

Optimal Hybrid Control For Structures *

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Abstract

A new method for integrated design of passive and active elements is presented. Rather than the existing qualitative selection of parameters for passive elements, a quantitative approach is proposed which finds optimal active and passive parameters with respect to an H_2/H_∞ performance requirement. This new approach automatically yields passive designs when the given performance limits are high enough, and active (hybrid) designs when the given performance constraints are stringent. Furthermore, our algorithm finds that the special performance requirement (the peak of the frequency response) which cannot be satisfied by any passive design. Hence, this paper shows how to determine WHEN is control required, rather than assuming a priori that it is or it is not required. A simple design method given herein yields any one of passive, active and hybrid designs, depending only on the level of the performance constraints that are specified in the statement of the problem.

1 Introduction

Some research effort in Japan and the USA are focused on control techniques to suppress the vibration of structures induced by earthquake, high winds and moving loads. Those techniques can be classified into passive control, active control and hybrid control, among other labels.

Passive control has been intensively used, including base isolation, friction dampers, passive bracing systems, tuned-mass dampers, visco-elastic dampers, etc. The advantage of passive control systems lies in their ability to absorb vibrational energy without the requirement of power or sensing, and their reliability and robustness (unconditionally stable system). However the passive devices are difficult to tune after the structure's construction, and in some cases (for example the tall structures), passive control is not sufficient to meet the performance requirement.

Active control uses external power and sensing to add damping or force to structures through feedback. The advantages of active control include the ability to control high order vibration modes, automatic tunability, working for stringent performance requirements. However the large power consumption and the question of actuator reliability in high loading and adverse operation conditions are factors preventing it from wide acceptance. The active control used in civil structure application has been studied intensively^{12,10,5}. Although the implementations of active control can be found in many aerospace structures, the implementations in civil structures are very recent¹¹. Much can be learned from the aerospace experience that will benefit the civil applications.

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Hybrid control systems combine passive and active control systems and overcome the weaknesses of both systems. The combination of base isolation and an active device is an example of hybrid control systems^{8,7}. The control effort often combats certain dynamics of the structure. These dynamics could have been made “easier to control” by making certain structural modifications through passive control. Only those dynamics which can not be accommodated by passive control are left for active control action. Hybrid control reduces the control force compared with active control and improves control effectiveness compared with passive control. This is the motivation for hybrid control.

The commonly used method in hybrid control is a cascade design procedure. The passive control is first designed by specifying devices or damper types and their parameters, then an active control algorithm is synthesized based on the augmented passive control system. It is a well-known fact that the structure and its control design problems are not independent⁹. Hence passive control is not independent of active control. Integrating passive parameter design with active control design will improve control effectiveness and energy consumption.

In this paper, we assume both the passive and active control devices are available for design. In our example, the passive devices are damper and stiffness devices. The active control devices could be an active brace system, tendon system and active mass drive system. We want to design passive parameters and active control algorithm such that the hybrid controlled system meets a stringent performance requirement. An ideal approach for this problem is to simultaneously design the passive and active parameters to optimize the performance index. However this approach is far from computationally tractable due to the complex nature of the optimization. The approach used here follows the philosophy introduced in Grigoriadis, et. al.⁴. Here in the first step a controller is designed for a set of nominal structured parameters. The closed loop system for this controller defines the desirable performance, even though the controller may be terribly unattractive (uses too much control energy, etc.). Then a second step in the design process optimizes (simultaneously) the structure and control parameters. This is a nonconvex (hard) optimization problem, but the trick that makes the method effective is to add a constraint to make this constrained optimization problem convex. This paper adds constraint to match the state space matrices of the involved transfer matrix whose frequency response peak needs to be limited. This method allows the mass, damping and stiffness matrices to contain free design parameters. The match constraint preserves the dynamic properties for the involved transfer function obtained in the first step which defined the “ideal” performance by an original controller. The variation of this design theme, which we have called “Optimal Mix of Passive and Active Control”, include different choice of control design criterion (that defines the “ideal” performance). Grigoriadis, et. al.⁴ proposes a computationally tractable iteration to perform the integrated design, where the active control energy is minimized subject to an upperbound constraint on output variances. When this solution yields zero control energy, then the design is completely passive. When the design yields nonzero control energy, then the selected performance upperbound cannot be achieved with any passive design. This paper also minimizes control energy, but constrains the peak value of the frequency response, instead of constraining the output variances.

This paper is organized as follows. Section 2 gives the mathematical description of the hybrid control problem, a brief discussion about H_2 and H_∞ norms and their upperbounds, and the mixed H_2/H_∞ control problem. In section 3, the optimal hybrid control problem for systems with equivalent features are studied. An iterative but convergent procedure for using the results in section 3 to general systems is studied in section 4. An example is included in section 5. Section 6 concludes the paper.

The following notations are used in this paper. $(\cdot)^T$, $(\cdot)^+$, $(\cdot)^{-1}$, $\text{tr}(\cdot)$ denote the transpose, Moore-Penrose generalized inverse, inverse and trace operations of a matrix (\cdot) respectively. A $n \times n$ unit matrix is denoted as $I_{n \times n}$. $\overset{\text{SSR}}{=}$ is short for *state space realization* of a transfer matrix. A positive definite matrix X is denoted as $X > 0$ and $X < 0$ is defined as $-X > 0$. $\text{vec}(\cdot)$ operator stacks the columns of a matrix. \otimes denotes the Kronecker product operation between two matrices.

2 Mathematical Description of Optimal Hybrid Control

For small motions, civil structures can be described by linear lumped parameter systems. Consider an initial designed structure, which is designed from some preliminary considerations

$$\begin{aligned} E_0(\dot{x} - B_1 w) &= A_0 x + B_2 u \\ z_1 &= \begin{bmatrix} x^T F_1^T & \dot{x}^T F_2^T \end{bmatrix}^T \\ z_2 &= C_2 x + D_{22} u \\ y &= \begin{bmatrix} x^T M_1^T & \dot{x}^T M_2^T \end{bmatrix}^T + D v \end{aligned} \quad (1)$$

where z_1 and z_2 are two kinds of performance variables which are used for two different performance requirements, y is the measurement, w is the external disturbance applied to the structure (e.g., the earthquake excitation), and v is the sensor noise. We have the following assumptions for this initial structure due to physical considerations

- (A1) The system (1) has independent measurements, i.e., M_1 and M_2 have full row rank.
- (A2) The system (1) has independent actuators, i.e., B_2 has full column rank.
- (A3) E_0 is inevitable.

Assume a structure parameter (passive control) p

$$p \triangleq \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}^T \quad (2)$$

which falls within an admissible set

$$\mathbf{P} \triangleq \left\{ \begin{bmatrix} p_1 & p_2 & \cdots & p_n \end{bmatrix}^T : p_i^- \leq p_i \leq p_i^+, i = 1, 2, \dots, n \right\}$$

modifies the initial structure (1) into the following structure

$$E(p)(\dot{x} - B_1 w) = A(p)x + B_2 u \quad (3)$$

where $E(\cdot)$ and $A(\cdot)$ are linear functions of the structure parameter (passive control) p of the following forms

$$E(p) = E_0 + \sum_{i=1}^p p_i E_i, \quad A(p) = A_0 + \sum_{i=1}^p p_i A_i.$$

We assume

- (A4) $E(p)$ is inevitable for all possible $p \in \mathbf{P}$.

Besides the structure parameter modification, the following active control K is applied to the initial structure (1)

$$\begin{aligned}\dot{x}_c &= A_c x_c + B_c y \\ u &= C_c x_c + D_c y\end{aligned}\tag{4}$$

We have the following assumptions for the system (1) and the control (4)

(A5) If $D_c \neq 0$, then $M_2 = 0, D = 0$.

(A6) If $D \neq 0, M_2 \neq 0$, then $D_c = 0$.

Notice that the assumptions (A5) and (A6) guarantee the finiteness of the H_2 norm from w to z_1 of the system defined later in (5).

The hybrid control system uses (2) and (4) to control the initial structure (1), the corresponding closed loop system is called the *hybrid closed loop system*, which is of the following form

$$\begin{aligned}\dot{x} &= A(K, p)x + B(K)w \\ z_1 &= C_1(K, p)x + D_1w \\ z_2 &= C_2(K)x\end{aligned}\tag{5}$$

where w denotes the disturbances applied to the closed loop system, x is the augmented state, i.e.

$$w = \begin{bmatrix} w \\ v \end{bmatrix}, \quad x = \begin{bmatrix} x \\ x_c \end{bmatrix}.$$

If (A5) holds, then the system matrices of the *hybrid closed loop system* (5) are

$$\begin{aligned}A(K, p) &= \begin{bmatrix} E(p)^{-1}(A(p) + B_2 D_c M_1) & E(p)^{-1} B_2 C_c \\ B_c M_1 & A_c \end{bmatrix} \\ B(K) &= \begin{bmatrix} B_1 & 0 \\ 0 & 0 \end{bmatrix} \\ C_1(K, p) &= \begin{bmatrix} \begin{bmatrix} F_1 & 0 \end{bmatrix} \\ F_2 A(K, p) \end{bmatrix} \\ D_1 &= \begin{bmatrix} 0 & 0 \\ F_2 \begin{bmatrix} B_1 \\ 0 \end{bmatrix} & 0 \end{bmatrix} \\ C_2(K) &= \begin{bmatrix} C_2 + D_c M_1 & D_{22} C_c \end{bmatrix}.\end{aligned}$$

If (A6) holds, then the system matrices of the *hybrid closed loop system* (5) can be written as

$$\begin{aligned}A(K, p) &= \begin{bmatrix} E(p)^{-1} A(p) & E(p)^{-1} B_2 C_c \\ \begin{bmatrix} B_c M_1 \\ M_2 E(p)^{-1} A(p) \end{bmatrix} & A_c + B_c \begin{bmatrix} 0 \\ M_2 E(p)^{-1} B_2 C_c \end{bmatrix} \end{bmatrix} \\ B(K) &= \begin{bmatrix} B_1 & 0 \\ B_c \begin{bmatrix} 0 \\ M_2 B_1 \end{bmatrix} & B_c D \end{bmatrix}\end{aligned}$$

$$\begin{aligned}
C_1(K, p) &= \begin{bmatrix} F_1 & 0 \\ F_2 E(p)^{-1} A(K, p) & F_2 E(p)^{-1} B_2 C_c \end{bmatrix} \\
D_1 &= \begin{bmatrix} 0 & 0 \\ F_2 B_1 & 0 \end{bmatrix} \\
C_2(K) &= \begin{bmatrix} C_2 & D_{22} C_c \end{bmatrix}.
\end{aligned}$$

For the *hybrid closed loop system* (5), denote the transfer matrix from w to z_1 as $T_1(K, p)$ and from w to z_2 as $T_2(K, p)$.

The H_∞ norm of $T_1(K, p)$ represents the peak magnitude (peak singular value) of the frequency response of $T_1(K, p)$, which is denoted as $\|T_1(K, p)\|_\infty$. $\|T_1(K, p)\|_\infty$ can also be used to denote the square root of the energy amplification factor of the response z_1 with respect to all possible inputs w

$$\|T_1(K, p)\|_\infty^2 = \max_w \left\{ \frac{\text{energy of } z_1}{\text{energy of } w} : w \text{ has nonzero but finite energy} \right\}$$

The H_2 norm of $T_2(K, p)$ is defined as the energy of the output z_2 with respect to a unit intensity white noise signal w .

The H_∞ norm of $T_1(K, p)$ does not exceed γ_1 if and only if there exists a $P = P^T > 0$ such that ¹

$$\begin{bmatrix} A(K, p)P + PA^T(K, p) & B(K) & PC^T(K, p) \\ B^T(K) & -I & D^T \\ C_1(K, p)P & D_1 & -\gamma_1^2 I \end{bmatrix} < 0. \quad (6)$$

The H_2 norm of $T_2(K, p)$ does not exceed $\gamma_2 > 0$ if and only if there exists a $Q = Q^T > 0$ such that ¹

$$\begin{bmatrix} A(K, p)Q + QA^T(K, p) & B(K) \\ B^T(K) & -I \end{bmatrix} < 0 \\ \text{tr}[C_2(K)QC_2^T(K)] < \gamma_2^2. \quad (7)$$

For the fixed parameter p , finding the active control K to satisfy (6) or (7) can be solved by the well-known H_∞ control or H_2 control theory (Notice that the assumption (A5) and (A6) guarantee that the H_2 norm of $T_2(K, p)$ of the *hybrid closed loop system* is finite). However, finding a controller K to simultaneously satisfy both (6) and (7) is an open problem and it may be intractable computationally. For computational tractability in the LMI framework, a single Lyapunov matrix $X \triangleq P = Q$ is sought in the above conditions ². We call this matrix X the H_2/H_∞ *common Lyapunov matrix*. This simplification leads to a performance upperbound for both H_2 and H_∞ norms of the *hybrid closed loop system* (5). Denote the corresponding upperbounds for $\|T_2(K, p)\|_2$ and $\|T_1(K, p)\|_\infty$ as

$$\overline{\|T_2(K, p)\|_2}, \quad \overline{\|T_1(K, p)\|_\infty}.$$

Instead of considering the exact H_2/H_∞ control problem stated in section 2, which can not be solved by existing methods, we consider the following well-studied problems.

H_2/H_∞ Optimal Active Control: For a fixed plant parameter p , solve for the active controller K from the following optimization problem

$$\gamma_a(p) = \min_K \{ \overline{\|T_2(K, p)\|_2} : \overline{\|T_1(K, p)\|_\infty} \leq \gamma_1 \}.$$

Theorem 2.1: For a fixed passive control p , the H_2/H_∞ Optimal Active Control can be transferred into a convex optimization problem. Hence the optimal value $\gamma_a(p)$ is the global minimal of the optimization.

Proof: See ².

The solution of the above problem can be obtained by using the LMI control toolbox ³.

If we want to find both the passive parameters and the active control parameters such that the performance defined in the above H_2/H_∞ Optimal Active Control problem is minimized, then we are considering the following problem.

H_2/H_∞ **Optimal Hybrid Control:** Simultaneously solve for the active controller K and the plant parameter p from the following optimization problem

$$\gamma_h = \min_{K,p} \{ \overline{\|T_2(K,p)\|_2} : \overline{\|T_1(K,p)\|_\infty} \leq \gamma_1 \}.$$

Remark: : If we choose the output variable z_2 as the active control variable u and the solution for H_2/H_∞ Optimal Hybrid Control problem achieves $\gamma_h = 0$, then this solution is a pure passive solution. In practice if γ_h is smaller enough, then we think that the corresponding solution is a passive solution.

In the following section, we solve the H_2/H_∞ Optimal Hybrid Control problem by constraining the system matrices of the transfer matrix $T_1(K,p)$ to be the same before and after the hybrid control.

3 Optimal Hybrid Control for Systems with Equivalent Features

For a given passive control $p \in \mathbf{P}$ and a given active control K , the transfer matrix from w to z_1 of (5) can be expressed as the following state space realization

$$T_1(K,p) \stackrel{\text{SSR}}{=} \left[\begin{array}{c|c} \mathbf{A}(K,p) & \mathbf{B}(K,p) \\ \hline \mathbf{C}_1(K) & \mathbf{D}_1 \end{array} \right], \quad (8)$$

meaning that

$$T_1(K,p) = \mathbf{D}_1 + \mathbf{C}_1(K)(sI - \mathbf{A}(K,p))^{-1}\mathbf{B}(K,p).$$

Given two active and passive control pairs (\tilde{K}, \tilde{p}) and (K, p) , $T_1(\tilde{K}, \tilde{p})$ is said to be *system equivalent* to $T_1(K, p)$ if those two have the same system matrices, i.e.

$$\begin{aligned} \mathbf{A}(\tilde{K}, \tilde{p}) &= \mathbf{A}(K, p) \\ \mathbf{B}(\tilde{K}) &= \mathbf{B}(K) \\ \mathbf{C}_1(\tilde{K}, \tilde{p}) &= \mathbf{C}_1(K, p). \end{aligned}$$

For the given active and passive control pair (K, p) , denote the whole class of *system equivalent* systems as $\mathbf{E}(K, p)$. Notice that any element in $\mathbf{E}(K, p)$ has the same H_∞ norm. In the following, without loss of generality, we consider the system equivalent class for $p = 0$ ($\mathbf{E}(K, 0)$), i.e., all transfer matrices $T_1(\tilde{K}, \tilde{p})$ whose system matrices are the same as $T_1(K, 0)$. We have the following result.

Lemma 3.1: Let K be given. Consider any given pair (\tilde{K}, p)

(i) If (A5) holds, then $T_1(\tilde{K}, p)$ belongs to $\mathbf{E}(K, 0)$ if and only if

$$\begin{aligned}\tilde{A}_c &= A_c, \quad \tilde{B}_c = B_c \\ B_2\tilde{C}_c &= B_2C_c + \sum_{i=1}^p E_i E_0^{-1} B_2 C_c p_i \\ B_2\tilde{D}_c M_1 &= B_2 D_c M_1 \\ &+ \sum_{i=1}^p [E_i E_0^{-1} (A_0 + B_2 D_c M_1) - A_i] p_i.\end{aligned}\tag{9}$$

(ii) If (A6) holds, then $T_1(\tilde{K}, p)$ belongs to $\mathbf{E}(K, 0)$ if and only if

$$\begin{aligned}\tilde{A}_c &= A_c, \quad \tilde{B}_c = B_c \\ B_2\tilde{C}_c &= B_2C_c + \sum_{i=1}^p E_i E_0^{-1} B_2 C_c p_i \\ \sum_{i=1}^p [E_i E_0^{-1} A_0 - A_i] p_i &= 0.\end{aligned}$$

Proof: See Lu and Skelton ⁶.

Remark: We are interested in a set of the closed loop systems $T_1(\tilde{K}, \tilde{p})$ whose H_∞ norms are bounded by the same γ_1 . However, this set is very complicated to characterize. The system equivalent class $\mathbf{E}(K, p)$ is a subset of this set.

The Optimal Hybrid Control over the equivalent class $\mathbf{E}(K, 0)$ for a given controller K is to solve for (\tilde{K}, p) from the following optimization problem

$$\gamma_h = \min_{\tilde{K}, p} \{ \overline{\|T_2(\tilde{K}, p)\|_2} : T_1(\tilde{K}, p) \text{ belongs to } \mathbf{E}(K, 0) \}.\tag{11}$$

The following theorem provides the solution for this optimization.

Theorem 3.2: For a given controller K of the form (4) or

$$K \stackrel{\text{SSR}}{=} \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c & D_c \end{array} \right]$$

and a H_∞ performance level $\gamma_1 > 0$, let X_0 be the H_2/H_∞ common Lyapunov matrix defined in section 2. Then the optimal solution over the output equivalent class $\mathbf{E}(K, 0)$ is a convex, constrained quadratic optimization problem with respect to the plant parameters p , and reduce specifically to solving the following optimization problem

$$\begin{aligned}\gamma_h &= \min && \text{tr}[a(p)^T (X_0 \otimes I) a(p)] \\ &\text{subject to} && p \in \mathbf{P}, \quad \hat{N}p = 0\end{aligned}\tag{12}$$

where

$$\begin{aligned}a(p) &= \text{vec}(W_0) + \hat{W}p \\ \hat{W} &= [\text{vec}(W_1) \quad \text{vec}(W_2) \quad \cdots \quad \text{vec}(W_n)] \\ \hat{N} &= [\text{vec}(N_1) \quad \text{vec}(N_2) \quad \cdots \quad \text{vec}(N_n)].\end{aligned}$$

If we denote

$$U_i = E_i E_0^{-1} B_2 C_c$$

W_i, N_i ($i = 1, 2, \dots, n$) can be computed from the following for different cases

(i) If (A5) holds, then

$$\begin{aligned} W_0 &= [C_2 + D_{22}D_cM_1 \quad D_{22}C_c] \\ W_i &= [D_{22}B_2^+V_iM_1^+M_1 \quad D_{22}B_2^+U_i] \\ N_i &= \begin{bmatrix} (I - B_2B_2^+)U_i \\ B_2B_2^+V_iM_1^+M_1 - V_i \end{bmatrix} \end{aligned}$$

with $V_i = E_iE_0^{-1}(A_0 + B_2D_cM_1) - A_i$.

Assume p_{opt} is the optimal solution of (12), then the optimal active controller K_{opt} has the following state space form

$$K_{\text{opt}} \stackrel{\text{SSR}}{=} \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c + B_2^+ \sum_{i=1}^n U_i p_{\text{opt}_i} & D_c + B_2^+ \sum_{i=1}^n V_i M_1^+ p_{\text{opt}_i} \end{array} \right].$$

(ii) If (A6) holds, then

$$\begin{aligned} W_0 &= \begin{bmatrix} C_2 & D_{22}C_c \end{bmatrix} \\ W_i &= \begin{bmatrix} 0 & D_{22}B_2^+U_i \end{bmatrix} \\ N_i &= \begin{bmatrix} (I - B_2B_2^+)U_i \\ E_iE_0^{-1}A_0 - A_i \end{bmatrix}. \end{aligned}$$

Assume p_{opt} is the optimal solution of (12), then the optimal active controller has the following state space form

$$K_{\text{opt}} \stackrel{\text{SSR}}{=} \left[\begin{array}{c|c} A_c & B_c \\ \hline C_c + B_2^+ \sum_{i=1}^n U_i p_{\text{opt}_i} & 0 \end{array} \right].$$

Proof: The proof can be done by using lemma 3.1 and some algebraic manipulations. Detail can be found in Lu and Skelton ⁶.

In the following section, the optimal hybrid control problem for general systems is solved by using the result in theorem 3.2 at one step of an iterative procedure.

4 Optimal Hybrid Control for General Systems

An algorithm for solving the Optimal Hybrid Control problem considered here could be summarized as follows. At each step, two tasks are performed. In the first task, the optimal performance is sought by solving the mixed H_2/H_∞ control problem for passive control fixed at the previous step; in the second task, an optimal hybrid control design is performed to match the system matrices of the previous step. Individually, each of those two tasks provides a global optimal solution, while the sequential combination of those two tasks will only provide a locally optimal solution. However the convergence of this sequential combination is guaranteed.

Iterative Algorithm

Step 1 Set $k=0$. Pick an initial passive control $p^k \in \mathbf{P}$ and formulate the state space system matrices as in (2).

The natural damping and stiffness might not be enough for the building to adequately suppress the vibrations caused by earthquakes. We are allowed to use hybrid control to control the vibration. The passive control can add damper and stiffness devices between floors and between the 1st floor and the ground, see (b) of Figure 1. Denote d_i, k_i as the damping and stiffness coefficient of the passive device between the i th floor and $(i-1)$ th floor of the building, and the passive control parameters are denoted by

$$p = [k_5, \dots, k_1, d_5, \dots, d_1]^T.$$

Hence the passive control system can be expressed in (3), where

$$E_0 = \begin{bmatrix} I & 0 \\ 0 & M_0 \end{bmatrix}, \quad A(p) = \begin{bmatrix} 0 & I \\ -K(p) - K_0 & -D(p) - D_0 \end{bmatrix}$$

where the damping matrix $D(p)$ and the stiffness matrix $K(p)$ can be written as the following linear functions of p

$$D(p) = \begin{bmatrix} d_5 & -d_5 & & & \\ -d_5 & d_5 + d_4 & -d_4 & & \\ & -d_4 & d_4 + d_3 & -d_3 & \\ & & -d_3 & d_3 + d_2 & -d_2 \\ & & & -d_2 & d_2 + d_1 \end{bmatrix}$$

$$K(p) = \begin{bmatrix} k_5 & -k_5 & & & \\ -k_5 & k_5 + k_4 & -k_4 & & \\ & -k_4 & k_4 + k_3 & -k_3 & \\ & & -k_3 & k_3 + k_2 & -k_2 \\ & & & -k_2 & k_2 + k_1 \end{bmatrix}.$$

Notice that q_i here denotes the displacement of the i -th floor relative to the ground.

The active control devices are chosen as tendon systems (Figure 2 (a)) or the active brace systems (Figure 2 (b)). A mathematical simplification is depicted in Figure 2 (c) where u_1, u_2, \dots, u_5 are the control variables.

Combining the passive modification as in Figure 1 and the active control as in Figure 2, we obtain a hybrid control system which has the form (5).

In the following discussion, all the physical parameters are normalized to simple numbers such that the discussion emphasizes the mechanism of the method. The system state is

$$x(t) = [q^T(t) \quad \dot{q}^T(t)]^T$$

where

$$q(t) = [q_5(t) \quad q_4(t) \cdots q_1(t)]^T$$

denotes the displacements relative to the ground of all the floors of the building and

$$B_1 = \begin{bmatrix} 0 \\ \mathbf{B}_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ \mathbf{B}_2 \end{bmatrix}$$

with

$$\mathbf{B}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{B}_2 = I_{5 \times 5}.$$

The hybrid control objective is to limit the interstory lateral drift and the absolute acceleration of each floor, and minimize the control energy. The interstory lateral drift is defined as

$$q_{is}(t) = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & 1 & -1 & \\ & & & 1 & -1 \end{bmatrix} q(t)$$

while the absolute acceleration vector of the building is

$$\ddot{q}_a(t) = \ddot{q}(t) + \mathbf{B}_1 \ddot{q}_g(t).$$

We choose $q_{is}(t)$ and $\ddot{q}_a(t)$ as H_∞ performance variables, i.e.

$$z_1(t) = [q_{is}^T(t) \quad \ddot{q}_a^T(t)]^T.$$

This is equivalent to limiting the peak value of the frequency response of $T_1(K, p)$ or the maximum energy of the output response z_1 with respect to all inputs w having unit energy bound.

The H_2 performance variable is chosen as the active control $u(t)$. Hence the mixed H_2/H_∞ control problem is to find a minimum energy control K for white noise earthquakes and at the same time limit the energy amplification factor associated with interstory drift and absolute accelerations. Hence the H_2/H_∞ Optimal Hybrid Control finds the passive control parameters k_i and d_i ($i = 1, 2, \dots, 5$) plus the active controller K to achieve the H_2/H_∞ performance.

Notice that the colored (non-white) earthquake excitation can also be cast in the frame work of this paper by some modification. Colored signals with known spectrum can be generated by sending a white noise to a linear filter (for example, the Kanai-Tajini spectrum earthquake signal). Hence for non-white earthquake signal $w(t) = \ddot{q}_a(t)$, we can find a filter $F(s)$ and a white noise $v(t)$ such that

$$w(s) = F(s)v(s).$$

Then in the mixed H_2/H_∞ control, instead of considering the transfer matrix $T_2(K, p)$ from w to z_2 , we should consider the transfer matrix from v to z_2 , i.e., $T_2(K, p)F(s)$.

If we impose a small performance bound on limiting the energy amplification factor in channel z_1 , the minimal active control energy might still be zero. If we minimize active control energy and at the same time bound energy amplification factor in channel z_1 tightly enough, then the minimal active control energy must not be zero. The sensor measurement here is taken as the displacement of each floor and the rates of the first, third and the fifth floors.

The initial structure parameters are

$$k_{i0} = 0.5, \quad d_{i0} = 0.01414, \quad m_i = 1, \quad i = 1, 2, \dots, 5.$$

Due to the implementation limitation, the stiffness k_i and the damping d_i of the passive control must fall within the physically realizable region. The following bounds characterize the admissible stiffness and damping

$$0 \leq k_i \leq 0.5, \quad 0 \leq d_i \leq 9d_{i0}, \quad i = 1, 2, \dots, 5,$$

We first study the passive control problem, i.e., we want to know the smallest γ_1 yielding passive design.

For $\gamma_1 = 1000$, the Optimal Passive Control leads to the following parameters

$$\begin{bmatrix} k_5 \\ k_4 \\ k_3 \\ k_2 \\ k_1 \end{bmatrix}^{\text{opt}} = \begin{bmatrix} 0.0054 \\ 0.0078 \\ 0.0096 \\ 0.0101 \\ 0.0088 \end{bmatrix}, \quad \begin{bmatrix} d_5 \\ d_4 \\ d_3 \\ d_2 \\ d_1 \end{bmatrix}^{\text{opt}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.1273 \end{bmatrix}$$

The distribution of the passive control along the floors is shown in Figure 3. In this case, the active control energy with respect to unit intensity white noise earthquake is smaller enough (less than 0.0002), we could think this as a passive control solution.

Now consider a tighter H_∞ bound. Let $\gamma_1 = 10$. Without passive control (i.e., the initial structure), a mixed H_2/H_∞ controller K^0 is first designed by solving the H_2/H_∞ Optimal Active Control problem. The active control energy of this controller with respect to a unit intensity white noise earthquake is

$$\|T_2(K^0, 0)\|_2^2 = 0.5694.$$

Now we add the hybrid control. For the convergence stopping criterion $\epsilon = 10^{-2}$, the algorithm proposed in section 4 converges after 98 iterations (see Figure 4). The optimal passive design are obtained as

$$\begin{bmatrix} k_5 \\ k_4 \\ k_3 \\ k_2 \\ k_1 \end{bmatrix}_{\text{opt}} = \begin{bmatrix} 0.2002 \\ 0.4292 \\ 0.5 \\ 0.5 \\ 0 \end{bmatrix}, \quad \begin{bmatrix} d_5 \\ d_4 \\ d_3 \\ d_2 \\ d_1 \end{bmatrix}_{\text{opt}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.1273 \end{bmatrix}$$

The distribution of the passive control parameters and active control energies along the floors of the building is shown in Figure 5. The corresponding active control energy with respect to a unit intensity white noise earthquake is

$$\|T_2(K, p_{\text{opt}})\|_2^2 = 0.3729$$

i.e., the hybrid control reduces active control energy by 35%, or say the control action of the 35% active control energy is performed by the passive damping and stiffness devices.

6 Conclusion

This paper provides an iterative procedure to find the optimal passive control parameters and the active control parameters. The performance used here is the so-called mixed H_2/H_∞ one. Since the H_∞ norm can be used to describe the system response energy amplification factor, the peak value of its frequency response, hence incorporating H_∞ norm performance in hybrid control is of practical significance. Also notice that a smaller H_∞ norm for the closed loop system implies the good robustness with respect to certain unmodeled dynamics in the system.

Our example here quantitatively shows that the optimal hybrid control for seismic excitation requires as small stiffness as possible and as large damping as possible at the building base. The large active control energy should be placed on the locations where the stiffnesses are small, and the small active control energy should be placed on the locations where the stiffnesses are large. Those are active controller configurations to save active control energy consumption and at the same time to achieve certain performance requirements together with the passive control. The

hybrid control reduces significantly the control power consumption (in our example, %35 energy reduction is achieved).

REFERENCES

1. Boyd, S., Ghaoui, L. E., Feron E., and Balakrishnan, V., *Linear Matrix Inequality in System and Control Theory*, SIAM, 1995.
2. Chilali, M. and Gahinet, P., “ H_∞ Design with Pole Placement Constraints: an LMI Approach”, *IEEE Transaction on Automatic Control*, 1996, pp. 358–367.
3. Gahinet, P., Nemirovski, A., Laub, A. J., and Chilali, M., *LMI Control Toolbox for Use with Matlab*, The Mathworks Inc., 1995.
4. Grigoriadis, K. M., Zhu, G., and Skelton, R. E., “Optimal Redesign of Linear Systems”, *ASME Journal of Dynamic Systems, Measurement and Control*, **118**, pp. 598–605.
5. Housner, G. W., Masri, S. F., and Chassiakos, A. G., *Proceedings of First World Conference on Structural Control*, International Association for Structural Control, Los Angeles, California, 1994.
6. Lu, J. and Skelton, R. E., “Mixed Passive/Active Control with H_2/H_∞ Performance in Structural Control”, *Internal Report*, 1996, Purdue University.
7. Nagarajaiah, S., Riley, M., and Reinhorn, A., “Control of Sliding Isolated Bridge with Absolute Acceleration Feedback”, *ASCE Journal of Engineering Mechanics*, **119**.
8. Reinhorn, A. M., Soong, T. T., and Wen, C. Y., “Base Isolated Structures with Active Control”, *Proceedings of ASME PVP Conference*, 1987, **PVP-127**, pp. 413–420.
9. Skelton, R. E., “Model Error Concept in Control Design”, *International Journal of Control*, 1989, **49**, pp. 1725–1753.
10. Soong, T. T., *Random Vibration of Mechanical and Structural Systems*, Longman Scientific and Technical, Essex, England, 1990.
11. Suzuki, T., Kageyama, M., Nobata, A., Inaba, S. and Yoshida, O., “Active Vibration Control System Installed in High-rise Building”, *Proceedings of the 1st World Congress on Structural Control*, 1994, **3**, pp. 3–11.
12. Yao, J. T. P., “Concept of Structural Control”, *Journal of the Structural Division, ASCE*, 1972, **98(ST7)**, pp. 1567–1574.

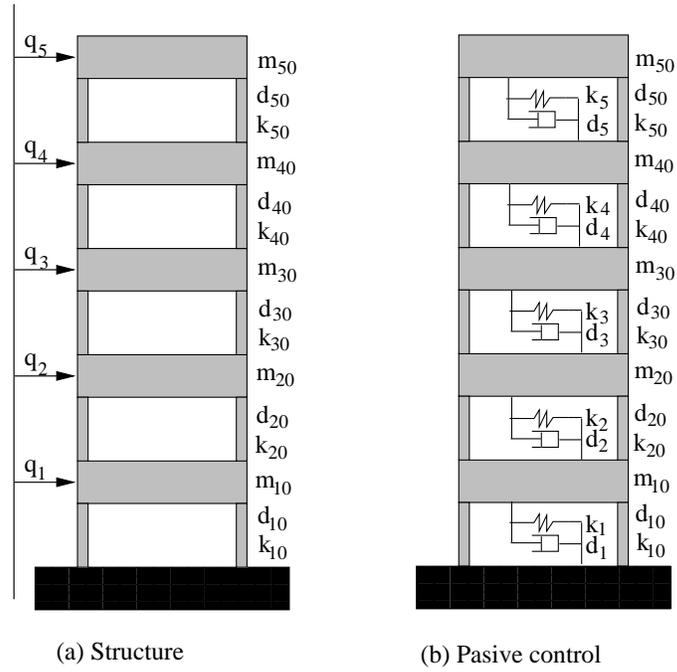
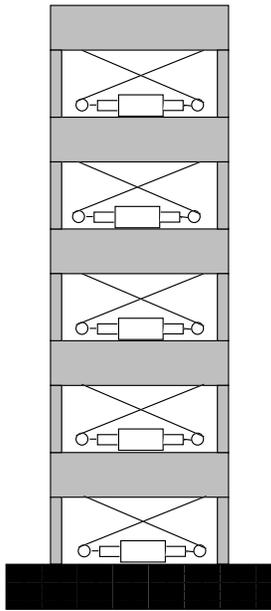
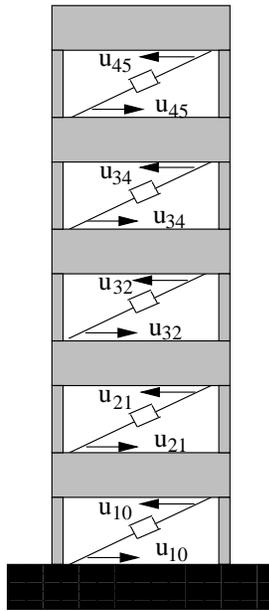


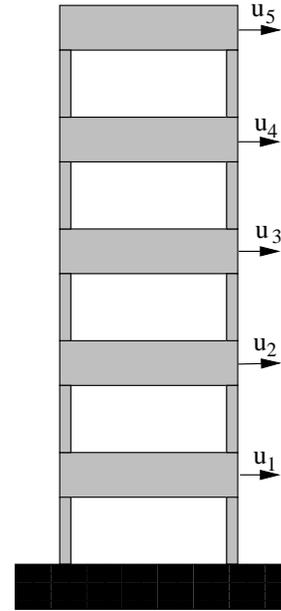
Figure 1: Passive control.



(a) Active tendon control



(b) Active brace control



(c) Mathematical Model

Figure 2: Active control.

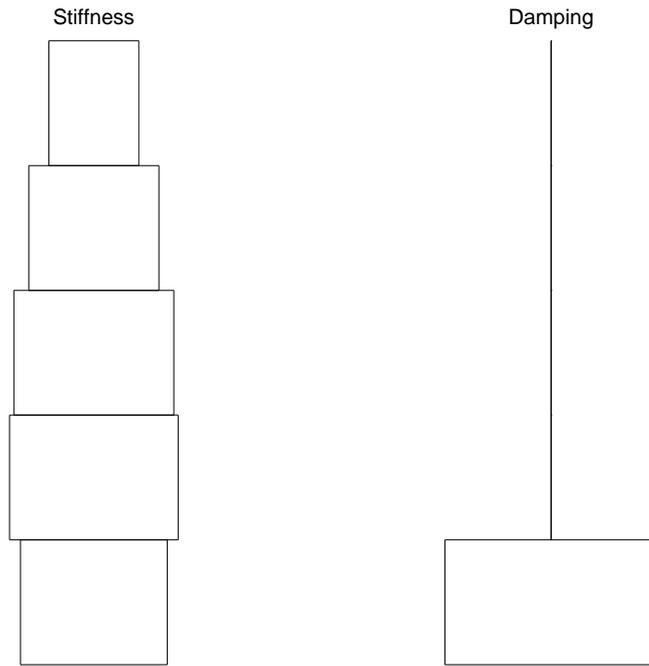


Figure 3: Distribution of the optimal passive control parameters (stiffness and damping) along the floors.

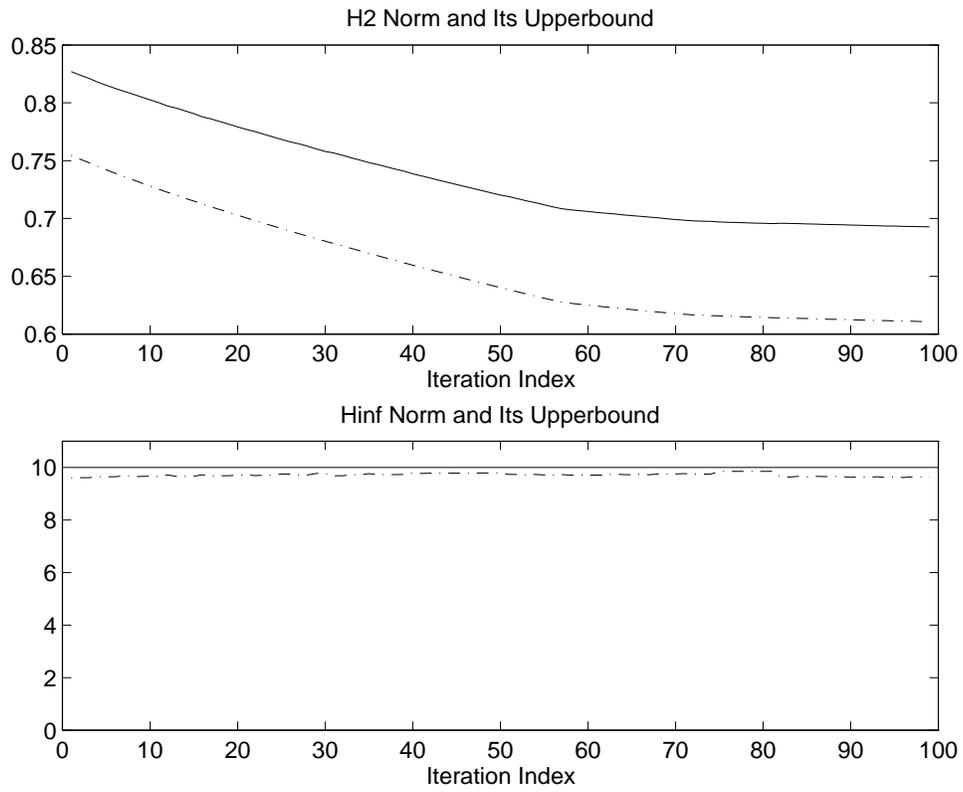


Figure 4: The iteration convergence study. The upper plot: the H_2 norm (dashed line) and its upperbound (solid line) converge after 98 iterations for a tolerance of $\epsilon = 10^{-2}$. The lower plot: the H_∞ norm (dashed line) and its upperbound (solid line).

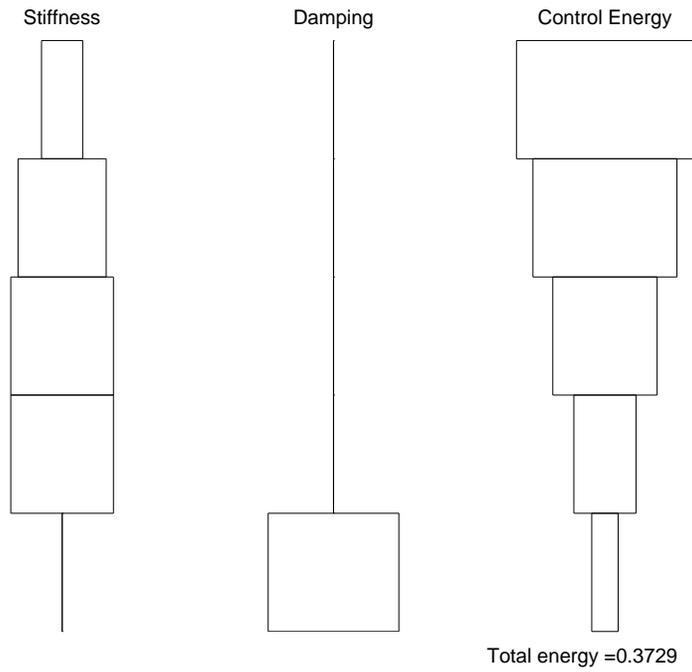


Figure 5: Distribution of the passive control parameters (stiffness and damping) and active control energy along the floors.