A Frequency-Adaptive Multi-Objective Suspension Control Strategy

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This paper studies a new FAMOS strategy for suspension control. FAMOS stands for frequency-adaptive multi-objective suspension. This strategy adjusts the control law based on certain frequency information and achieves a balanced ride and handling performance. It contains a road profile identifier, several multi-objective control laws which optimize a mixed H_2/H_∞ performance index based on different performance preferences, and an adaptive law based on the frequency contents estimated from the identified road profile. The strategy is applied to a quarter car suspension control and the simulation results show that the achieved performance is better than many existing results.
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1 Introduction

The current advance in microprocessors boosts applications of electronically controlled systems in automotive vehicles. Such electronically controlled systems include vehicle dynamics control, anti-lock brake system, electronic throttle control, steer by wire, brake by wire, all wheel-drive, power-train control, controlled suspension, etc. In this paper we focus on controlled suspensions.

Controlled suspensions for transportation systems have been studied for decades in both industry and the scientific community. Researchers have been using controlled suspensions as one of the most popular examples to test newly developed control strategies. The majority of the studies are usually based on a simple quarter car model (see Refs. [4,6,7,9,11,13,14]), although more complex models have been studied (see Ref. [8] for references). While quarter car model is not particularly realistic, it continues providing a convenient framework for investigating new approaches. This paper focuses on a quarter car model, however the strategy is applicable to a full car model. For example, the multi-objective control strategy used here for a quarter model has been proved suitable for full car suspension control [10]. The common objectives of suspensions include: transmitting to passengers as small road profile as possible (ride comfort or body performance) and keeping the wheels following the road surface (handling performance or wheel performance). Although the objectives are well known, many studies in controlled suspensions are biased toward the body performance. It has been found that a body-performance-oriented controlled suspension would not help much for achieving wheel performance, and the wheel-performance-oriented controlled suspension will surely worsen the body performance. Therefore a conflicting performance requirement exists. Since multi-objective optimal control approaches are good at achieving trade-off between conflicting requirements, it is natural to apply them to controlled suspensions. It is the intention of this paper to use the so-called *mixed H_2/H_∞ control* to achieve a balanced ride and handling performance.

While the aforementioned mixed H_2/H_∞ control delivers a trade-off performance for ride and handling, its achievable performance is limited due to performance limitations [13]. If the condition of "a single linear controller" is broken, the achieved performance can be further improved. This paper proposed a preliminary result using frequency-adaptive control strategy. This strategy first estimates the road profile, then based on the frequency contents of the estimated road profile, the suspension controller is adjusted toward different multi-objective controllers. Such a control strategy is called a *frequency-adaptive multi-objective suspension*, short to FAMOS. Notice that in Ref. [4], a road-adaptive linear parameter varying control scheme is introduced and the road information is switched to three different levels based solely on the magnitude of the suspension height. In the current paper we prove that the road profile is related to the suspension height in a rather complicated dynamic fashion. Since the approach in Ref. [4] is not intended for adaptation based on road frequency contents, the achieved body performance in high frequency is even worse than the passive suspension (see Fig. 5 in [4]). The approach studied here can avoid such limitations (for example, achieving low frequency performance at the expense of high frequency performance).

The paper is organized as the follows. Section II contains a discussion above the vehicle state and road profile estimation using a suspension height sensor. The suspension control performance requirements are discussed in Sec. III. Section IV is devoted to FAMOS (frequency-adaptive multi-objective suspension) strategy. This strategy is further illustrated in a numerical example in Sec. V. Section VI offers our conclusions.

2 Vehicle and Road State Sensing

Many control schemes use vehicle states for feedbacks. However, not all of the vehicle states are available. It is of great interest to use limited sensor signals for predicting and estimating the unavailable vehicle and road states. The vehicle states include the body heave, roll and pitch angles, and the tire deflections. The road states contain road profiles which characterize the vertical variations of the road surface at tire contact patches, the road surface friction and the road bank and inclination. A quarter car model is studied in this paper, therefore the involved states are the body heave, the tire deflection and the road profile. Notice that the results presented here can be extended to a full-car model. In this paper a suspension height sensor is used.

Let us consider the quarter car model shown in Fig. 1, where z_b is the heave, z_w the wheel displacement, w the road profile, $z_{\rm sh}$ the suspension height, $z_{\rm td}$ the tire deflection and f the controlled suspension force. The terms m_b and m_w are the sprung and un-sprung masses, k_s and c_s are the passive suspension stiffness and damping, and k_t is the tire vertical stiffness.

Suspension Actuation Force Realization. The controlled suspension force generated from the suspension actuator is different from the demand force computed from the control algorithm due to actuator dynamics. That is, there is a dynamic mapping $m(\cdot;\cdot)$ such that $f=m(t;z_{\rm sh},\dot{z}_{\rm sh},v)$, where v is an electronic signal (for example, a voltage or a current) which serves as the suspension actuator command. If the demand force calculated from the suspension control law is u, the corresponding actuator command v might be readily solved as $v=m^{-1}(t;u,z_{\rm sh},\dot{z}_{\rm sh})$, where $m^{-1}(\cdot;\cdot)$ is the inverse dynamic mapping of $m(\cdot;\cdot)$. If the inverse mapping is not available, a local controller (usually a nonlinear one) can be designed such that the true actuator force follows the demand one.

Since the focus of the FAMOS strategy presented here is not on the actuation, we will assume there exist an actuation technology and strategy capable of producing the demand force. It is known in the literature that significant challenges exist in the generation of arbitrary forces for certain types of actuation system [2] and experimental results along this line are scarce. However, imple-

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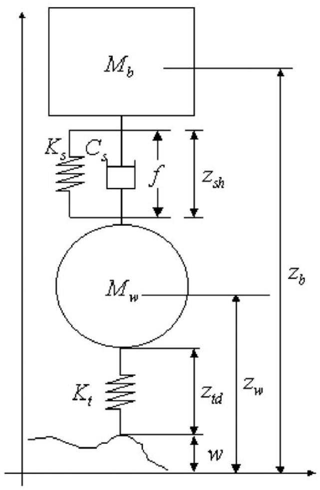

Fig. 1 A quarter car vehicle model

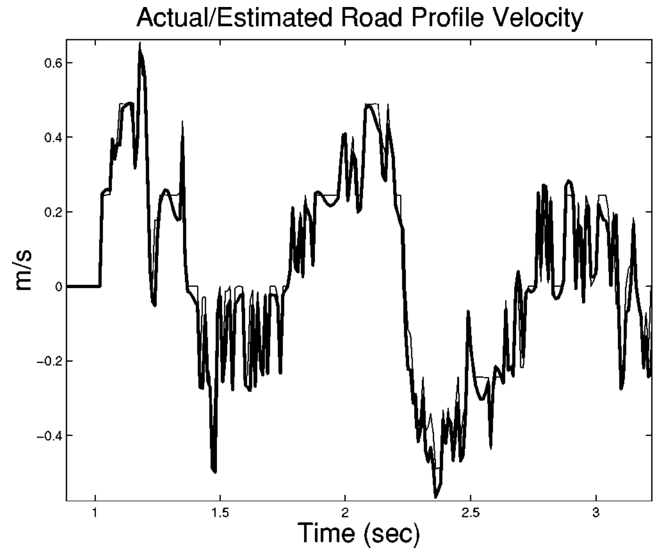

Fig. 2 Comparison between the actual (thin line) and estimated (thick line) road profile velocity $\hat{w}$

mentation results such as Ref. [1] have shown that it may indeed be possible thereby validating a focus on an outer loop force generation strategy such as FAMOS. New technologies such as high performance valve, magneto-rheological damper, etc., have also shown promises in alleviating the actuation limitations found in traditional hydraulically actuated suspensions.

Road Profile Estimator. The road adaptive control strategy needs first to know the frequency contents in the road profile. In the sequential discussion, the following constants are used:

$$\omega_1=\sqrt{\frac{k_s}{m_b}},\quad \zeta_1=\frac{c_s}{2\sqrt{k_s m_b}},\quad \omega_2=\sqrt{\frac{k_t}{m_b+m_w}},\quad \omega_3=\sqrt{\frac{k_t}{m_w}},$$

where ω_1 is called the body frequency and ω_3 is close to the wheel hop frequency.

The acceleration $\ddot{w}$ of the road profile can be uniquely determined from the suspension height z_{sh} and suspension control force f. That is, there exist two transfer functions T_1 and T_2 such that

$$\ddot{w}=T_1(s)z_{\mathrm{sh}}+T_2(s)f, \tag{1}$$

where $T_1(s)$ and $T_2(s)$ can be found as in the following:

$$T_1(s)=-\left[\frac{s^4}{\omega_3^2}+2\zeta_1\omega_1\frac{s^3}{\omega_2^2}+\left(\frac{\omega_1^2}{\omega_2^2}+1\right)s^2+2\zeta_1\omega_1 s+\omega_1^2\right],$$

$$T_2(s)=\frac{1}{m_b}\left(1+\frac{s^2}{\omega_3^2}\right).$$

In order to obtain the road profile from Eq. (1), a double integration is involved. Since pure integration is not practically useful due to potential sensor drifts and numerical errors, the following high-pass filtered integration is used in this paper:

$$\frac{s^2}{(s^2+2\zeta_0\omega_0 s+\omega_0^2)^2}, \tag{2}$$

where the cutoff frequency $\omega_0 \ll \omega_1$ and ζ_0 is the damping coefficient. Applying Eq. (2) to Eq. (1), the estimation $\hat{w}$ of the road profile can be computed as

$$\hat{w}=\hat{T}_1(s)z_{\mathrm{sh}}+\hat{T}_2(s)f, \tag{3}$$

where $\hat{T}_1(s)$ and $\hat{T}_2(s)$ are the following transfer functions:

$$\hat{T}_1(s)=-\frac{\frac{s^4}{\omega_3^2}+2\zeta_1\omega_1\frac{s^3}{\omega_2^2}+\left(\frac{\omega_1^2}{\omega_2^2}+1\right)s^2+2\zeta_1\omega_1 s+\omega_1^2}{(s^2+2\zeta_0\omega_0 s+\omega_0^2)^2},$$

$$\hat{T}_2(s)=\frac{\left(1+\frac{s^2}{\omega_3^2}\right)}{m_b(s^2+2\zeta_0\omega_0 s+\omega_0^2)^2}.$$

The road profile velocity can be obtained by differentiating $\hat{w}$ together with a low-pass filter. Figure 2 shows a comparison between the actual and the estimated road profile velocity.

Vehicle State Estimator. The tire deflection z_{td} can be calculated through the following relationship without using an integration:

$$\hat{z}_{\mathrm{td}}=\frac{1}{\omega_3^2}\ddot{z}_{\mathrm{sh}}+2\frac{\zeta_1\omega_1}{\omega_2^2}\dot{z}_{\mathrm{sh}}+\frac{\omega_1^2}{\omega_2^2}z_{\mathrm{sh}}-\frac{f}{m_b\omega_2^2}. \tag{4}$$

Based on the suspension height sensor and the estimated variables, the vehicle heave can be estimated as

$$\hat{z}_b=z_{\mathrm{sh}}+\hat{z}_{\mathrm{td}}+\hat{w}.$$

The heave and tire deflection velocities can be calculated through differentiating the corresponding estimates.

In conclusion, all the involved states (including the vehicle body states, the wheel states and the road states) in a quarter car model can be solely estimated from the suspension height sensor and the controlled suspension force.

3 Controlled Suspension Performances

Although all control strategies can be applied to suspension controls, one needs to be cautious about what can be achieved and what should be achieved before a method is chosen. The performance limitations and performance measure are among the first things to be considered. Although many papers have dealt with several different control strategies, far few have examined achievable performance in the form to be explained below. Reference [14] showed that there is an invariant point in the frequency response for body acceleration and emphasizing road handling performance will cause deterioration in ride performance. This simply implies that H_∞ control strategy might not be proper for achieving ride performance. This has been clearly shown in Ref. [3] where an H_∞ control does not perform well in comparison with an linear quadratic Gaussian control. From a frequency response point of view, Ref. [13] shows that the involved frequency response cannot be arbitrarily shaped. For body acceleration, reducing magnitude in low frequencies is at the expense of increasing magnitude in high frequencies. Generally speaking, the body performance and the wheel performance are conflicting objectives. In the following two subsections, the body and wheel performances will be discussed in detail.

Body Regulation Performance. Ride comfort requires that the car body achieves small acceleration levels. Ideally we want to achieve zero body accelerations or perfect isolation of the body from the road profiles. Realistically, we want to minimize the magnitude of the heave, pitch and roll accelerations of the car body with respect to typical road profiles. This should be called body regulation performance (BRP).

On the other hand, what really affects a human body's comfort is the vibration frequencies felt by a passenger riding in the vehicle. The human body is proven to be very sensitive to the vibrations of frequencies around 1 Hz. This frequency region is called the *body frequency region*. Hence one of the major tasks in suspension control is to isolate the body from around 1 Hz road profiles.

Certain body parts like eyeballs, stomach, etc., have natural frequencies within 4–8 Hz, which is called the *middle frequency region*. Their vibrations will be excited if the road profiles have frequencies within this region. Therefore the controlled suspension also needs to attenuate body vibrations within the middle frequency region.

The next frequency region of interest is the so-called *harshness frequency region* where high frequency vibrations are induced. The high frequency vibrations could cause noise and uneasy feeling of the passengers. Hence the controlled suspension must also keep the body isolated as much as possible from the harshness frequencies.

In summary, the body accelerations should be attenuated for all frequency regions in order to achieve a desired BRP. Attenuating vehicle body vibrations within certain specific frequency regions but failing to do so in other frequency regions is not considered as a desired performance in this paper.

Let r denote the road profile, which could be any of three types of

$$r=w, \quad r=[w \ \ \dot{w}]^T, \quad r=[w \ \ \dot{w} \ \ \ddot{w}]^T.$$

Denote $T_{\mathrm{rb}}(P,C)$ as the transfer function from the road input r to the body acceleration $\ddot{z}_b$ and $T_{\mathrm{rb}}(P,C;j\omega)$ as its frequency response at frequency ω. Mathematically the above discussion can be summarized as the following problem.

Problem 1 (BRP Control). *Find a controller C such that for any frequency* $\omega\in[0,\infty)$, *the magnitude of* $T_{\mathrm{rb}}(P,C;j\omega)$ *is kept as small as possible and is less than the magnitude of* $T_{\mathrm{rb}}(P,0;j\omega)$ (*for passive suspension*). *That is, find a controller C such that* $\forall\omega\in[0,\infty)$

$$|T_{\mathrm{rb}}(P,C;j\omega)| \ \text{ is minimized} \tag{5}$$

and $|T_{\mathrm{rb}}(P,C;j\omega)|\leq|T_{\mathrm{rb}}(P,0;j\omega)|$.

Notice that the optimization in Problem 1 is somewhat similar to the so-called model predictive control where the control parameters are optimized at each time instant. Since the frequency-adaptive BRP control parameters are optimized at each frequency, we might call it *Frequency Domain Model Predictive Control*. There are no appropriate approaches for solving this problem at this moment and further study is needed. In order to form a solvable problem closely related to the performance requirement, variations from Problem 1 will be studied instead.

Wheel Following Performance. The road holding capability of wheels implies that the wheels follow the road profiles well. That is, the wheel vertical accelerations should not be kept at zero if the road profiles have nonzero accelerations. Such a wheel performance implies that the wheel normal forces are kept as constant as possible, or say the tire deflections are kept as constant as possible. This desired wheel behavior is called wheel following performance (WFP).

Since the road profile is usually white noise process, its spectrum is flat regardless of the frequency. Hence WFP implies that the tire deflection frequency response needs to be kept as constant as possible. That is, WFP can also be characterized through the frequency responses of the involved transfer functions.

Denote $T_{\mathrm{rw}}(P,C)$ as the transfer function from the road input r to the tire deflection z_{td} and $T_{\mathrm{rw}}(P,C;j\omega)$ as its frequency response at frequency ω. Mathematically WFP can be summarized as the following problem.

Problem 2 (WFP Control). *Find a controller C such that the magnitude of* $T_{\mathrm{rw}}(P,C;j\omega)$ *is as constant as possible for* ω *in the interested frequency region* Ω_w, *and its peak magnitude at the frequency region* Ω_w *must be less than that of the same transfer function for passive suspension. That is, find a controller C such that for* $\forall\epsilon>0$ *there exists a constant* $\beta>0$ *such that*

$$\sup_{\omega\in\Omega_w}|T_{\mathrm{rw}}(P,C;j\omega)-\beta|\leq\epsilon \tag{6}$$

and at the same time the following is true:

$$\sup_{\omega\in\Omega_w}|T_{\mathrm{rw}}(P,C;j\omega)|\leq\sup_{\omega\in\Omega_w}|T_{\mathrm{rw}}(P,0;j\omega)|, \tag{7}$$

where Ω_w *denotes the wheel frequency region around the wheel hop frequency*.

Notice that the above problem is similar to the frequency domain model predictive control setting discussed in the last subsection. Due to the fact that there is no method available at this moment to solve it, a variation from this problem but approximately reflecting the original performance requirement will be the focus of this paper.

4 FAMOS Strategy

The suspension control law is required to achieve good BRP and WFP simultaneously. Hence the individual problems discussed in the last section need to be combined together.

Problem 3 (BRP/WFP Control). *Find a controller C such that* $\forall\omega\in[0,\infty)$

$$|T_{\mathrm{rb}}(P,C;j\omega)| \ \text{ is minimized} \tag{8}$$

and at the same time the following are satisfied:

$$|T_{\mathrm{rb}}(P,C;j\omega)|\leq|T_{\mathrm{rb}}(P,0;j\omega)|$$

$$\sup_{\omega\in\Omega_w}|T_{\mathrm{rw}}(P,C;j\omega)-\beta|\leq\epsilon$$

$$\sup_{\omega\in\Omega_w}|T_{\mathrm{rw}}(P,C;j\omega)|\leq\sup_{\omega}|T_{\mathrm{rw}}(P,0;j\omega)|.$$

Problem 3 is also a constrained frequency domain optimization and its solution is frequency dependent in nature. One advantage of using such a frequency-dependent controller is that the performance limitation shown in Ref. [13] can be alleviated or removed. The actual implementation of this frequency-dependent controller might involve (i) building a fairly accurate frequency domain model; (ii) solving a highly nonlinear algebraic equation at each frequency in real time; (iii) estimating the road profile; (iv) generating the road profile frequency contents from the road profile. Since the road profile can be estimated from sensor signals (as shown in Section II), (iii) can be eliminated. However (i), (ii) and (iv) are directly related to the success of this strategy.

Although (i) and (ii) need to be further pursued, this paper proposes a preliminary result which adjusts the final control to several different fixed control laws based on limited numbers of frequency regions. Those fixed control laws achieve certain BRP and WFP based on performance preferences. One of those fixed controllers is the best balanced controller for achieving BRP and WFP trade-off. In cases where the road profile contains multiple frequencies within multiple frequency regions, this best balanced multi-objective control law becomes the last defense for vehicle performance.

Therefore, one of the building blocks for FAMOS is to find multi-objective controllers based on trade-off between BRP and WFP. The following single control problem is of interest, which is a conservative variation of Problem 3.

Problem 4 (Multi-objective H_2/H_∞ BRP/WFP Control). *For a given $\gamma<1$, find the suspension control law C to solve the following optimization*:

$$\min\{\|T_{\text{rb}}(P,C)T_{\text{rb}}^{-1}(P,0)\|_2 : \|T_{\text{rw}}(P,C)\|_\infty \leq \gamma \sup_\omega |T_{\text{rw}}(P,0;j\omega)|\}. \tag{9}$$

Notice that Eq. (9) is still open. However there is a computable approximation to it, which is called a mixed H_2/H_∞ control problem (see Ref. [12] and reference therein.)

A Mixed H_2/H_∞ Multi-Objective Control. Consider the following state space description of a quarter car P

$$\begin{aligned}\dot{x}&=Ax+B_1r+B_2u,\\ y=\dot{z}_{\text{sh}}&=C_yx+D_{y1}r+D_{y2}u,\\ z_\infty=z_{\text{td}}&=C_\infty x+D_{\infty1}r+D_{\infty2}u,\\ z_2&=C_2x+D_{22}u,\end{aligned} \tag{10}$$

where u denotes the demand suspension control force, y the suspension height derivative (measurement).

We want to find a control algorithm C of the following form:

$$\dot{x}_c=A_cx_c+B_cy, \quad u=C_cx_c \tag{11}$$

to limit the peak frequency response (measured by the so-called H_∞ norm) of the transfer function for output z_∞ and to minimize the H_2 norm of the transfer function for output z_2 ($\dot{z}_b$ or $\ddot{z}_b$).

Using the controller C, the closed loop system transfer function from r to z_2 is denoted as $T_2(P,C)$ and from r to z_∞ is denoted as $T_\infty(P,C)$. Instead of finding a C to solve a problem similar to problem 4, an upper bound approach is considered. Using the method in Ref. [5], the corresponding upper bounds for $\|T_2(P,C)\|_2$ and $\|T_\infty(G,C)\|_\infty$ are denoted as $\overline{\|T_2(P,C)\|}_2$, $\overline{\|T_\infty(P,C)\|}_\infty$. The recently well-studied mixed H_2/H_∞ control theory finds controllers such that those upper bounds are optimized or constrained.

Problem 5 (Mixed H_2/H_∞ BRP/WFP Control). *For a given quarter car state space description P, find the suspension control law to solve the following optimization for a given performance level $\gamma<1$*:

$$J=\min_C\{\overline{\|T_2(P,C)\|}_2 : \overline{\|T_\infty(P,C)\|}_\infty \leq \gamma\|T_\infty(P,0)\|_\infty\}. \tag{12}$$

By using the linear matrix inequality solver (for example, the linear matrix inequality control toolbox in Matlab), the solution for Eq. (12) can be easily found [5]. It is a convex optimization problem and the controller is globally optimal. Notice that γ shapes the performance of the final closed loop system in a sense that small γ tries to achieve WFP, while large γ tries to achieve BRP. A properly chosen γ could achieve a balanced BRP and WFP.

B FAMOS Control Law. As in the last subsection, several multi-objective controllers can be found based on different performance preferences. The achieved performances need further improvement due to performance limitations. As shown in Ref. [13] a good BRP in the low frequency region is at the expense of deteriorating BRP in the high frequency region if a single linear control is used. Similar limitations exist for WFP. The performance limitation can be alleviated or eliminated if the control is not limited to a single linear one. The road frequency-adaptive controller studied here switches among a set of controls based on the frequency contents in the road profile.

The success of the above switching depends on whether the frequency contents of a signal can be identified. The following is a summary of a frequency identification method using the first order zero-crossing algorithm:

$$t_{\text{zc}_{k+1}}=t_{\text{zc}_k}+\Delta T$$

if $(y_ky_{k+1}<0$ or $y_{k+1}=0)$

{

if $(|y_{k+1}-y_k|>\epsilon)$

{

$$dt_{\text{zc}_{k+1}}=-\Delta T\frac{y_k}{(y_{k+1}-y_k)};$$

}

else

{

$$dt_{\text{zck}+1}=-\frac{1}{2}\Delta T;$$

}

$$\omega_{k+1}=\frac{1}{(t_{\text{zc}_{k+1}}+dt_{\text{zc}_{k+1}})},$$

$$t_{\text{zc}_{k+1}}=\Delta T-dt_{\text{zc}_{k+1}},$$

}

where ΔT is the sampling time for FAMOS, ω_{k+1} is the current frequency of the signal, y_k could be any of the filtered estimated road profile derivative passing through appropriate bandpass filters. In order to achieve robust results, an averaging process based on the above algorithm is used.

Let us define the following frequency regions:

$$\Omega_b=\text{body frequency region, less than 4 Hz,}$$

$$\Omega_m=\text{middle frequency region, 4–8 Hz,}$$

$$\Omega_w=\text{wheel frequency region, 8–12 Hz,}$$

$$\Omega_h=\text{harshness frequency region, above 12 Hz.}$$

Frequency-adaptive Multi-objective Suspension (FAMOS) Strategy. *Let C^i solve multi-objective control Problem 5 and achieve desired performance at the frequency region Ω_i for $i=b$, m, w, h. Let C^{bw} be the balanced controller to achieve the best trade-off between BRP and WFP. If Ω_{rfc} is the estimated road frequency contents calculated from the estimated road profile velocity $\dot{\hat{w}}$, then a FAMOS controller $C_{\text{FAMOS}}\in C_\Omega$ (the set of controllers whose parameters are frequency dependent) is defined as the following*:

$$C_{\text{FAMOS}}=s_b(\Omega_{\text{rfc}}\cap\Omega_b)C^b+s_m(\Omega_{\text{rfc}}\cap\Omega_m)C^m+s_w(\Omega_{\text{rfc}}\cap\Omega_w)C^w+s_h(\Omega_{\text{rfc}}\cap\Omega_h)C^h$$

if Ω_{rfc} is a subset of any of the four regions Ω_b, Ω_m, Ω_w and Ω_h. If in case Ω_{rfc} contains frequencies falling within multiple regions of Ω_b, Ω_m, Ω_w and Ω_h, then

$$C_{\text{FAMOS}}=C^{\text{bw}},$$

where $s_i(\cdot)$ for $i=b$, m, w, h is a frequency-dependent scaling.

One set of the frequency-dependent scalings is shown in Fig. 3. This is a fuzzy-logic-like scheme. Other choices of the frequency-dependent scalings are possible. In this paper we focus on this

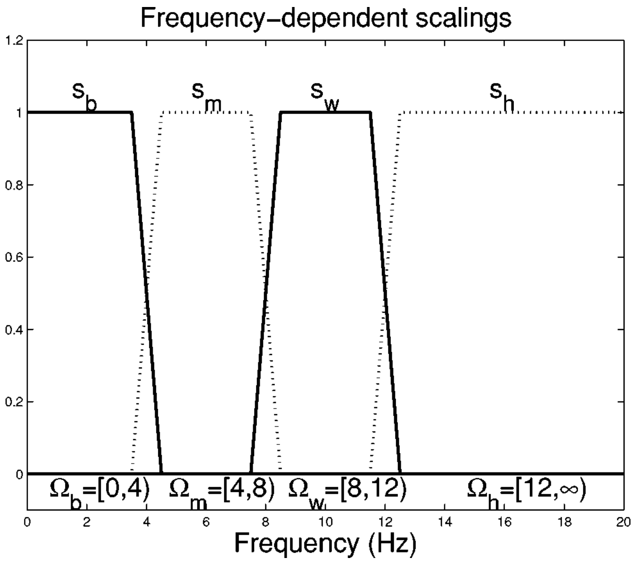

Fig. 3 The frequency-dependent scalings for FAMOS

fuzzy-logic-like scheme. The sum is equal to the corresponding individual controller if the frequency contents of the road profile are not within the region where two neighbor scales are nonzero. If the road frequency contents falls within such regions where both the neighbor scales are nonzero, the sum of those scales is equal to one. Hence the theoretic background for the above summing scheme is the same as the one used in fuzzy-logic control.

5 Numerical Examples

Now let us use an example to illustrate FAMOS strategy. The quarter car vehicle has the parameters: $m_b = 396\,(\mathrm{kg})$, $m_w = 61\,(\mathrm{kg})$, $k_s = 34{,}500\,(\mathrm{N/m})$, $c_s = 300\,(\mathrm{Ns/m})$, $k_t = 220{,}000\,(\mathrm{N/m})$. The following system matrices for the state space description of the quarter car model are used for controller design:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ -\dfrac{k_s}{m_b} & 0 & 0 & -\dfrac{c_s}{m_b} \\ -\left(\dfrac{k_s}{m_b} + \dfrac{k_s}{m_w}\right) & \dfrac{k_t}{m_w} & 0 & -\left(\dfrac{c_s}{m_b} + \dfrac{c_s}{m_w}\right) \end{bmatrix},$$

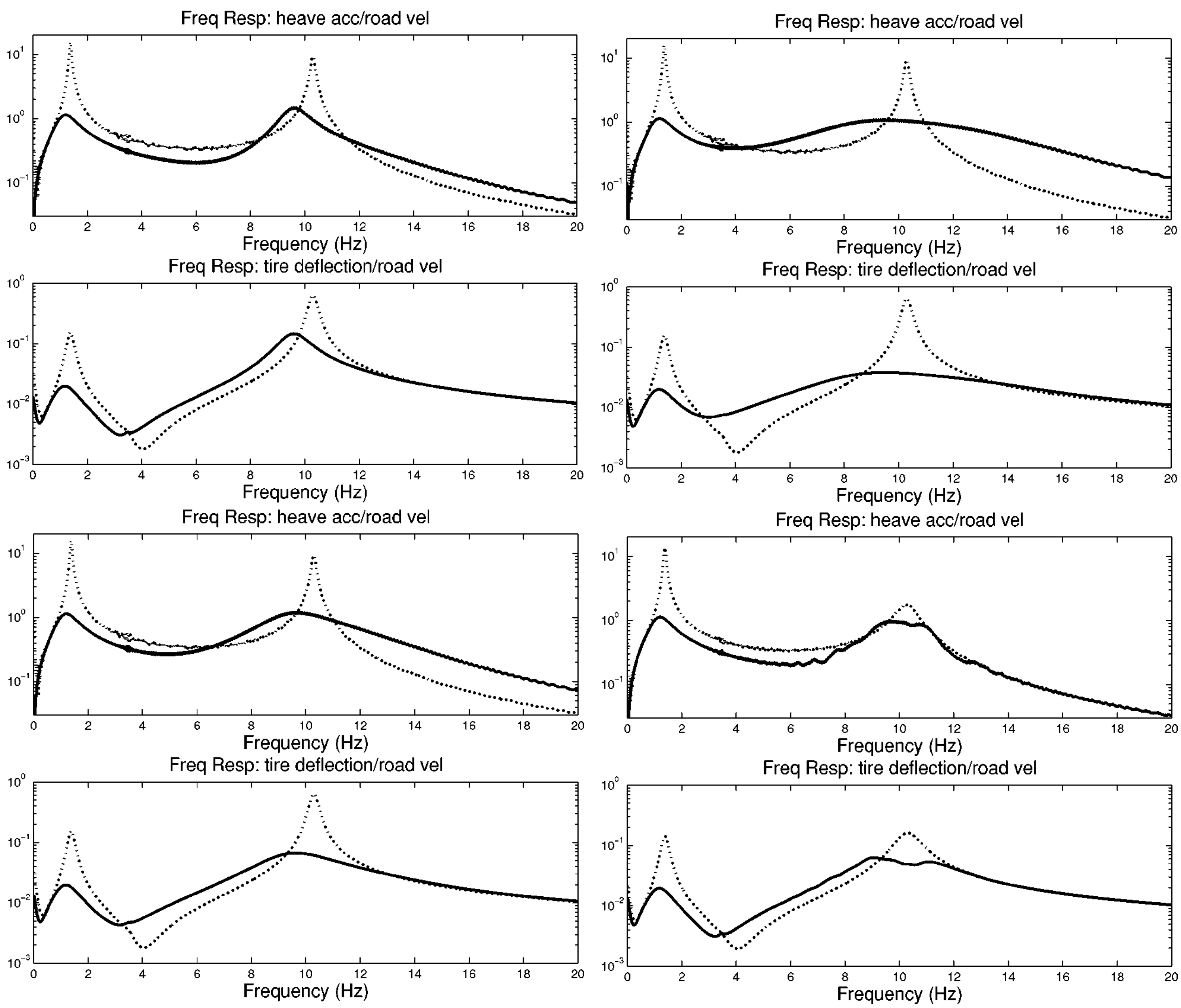

Fig. 4 Frequency responses. Dotted line: passive; solid line: controlled. Top-left 2 for BRP control; top-right 2 for WFP control; bottom-left 2 for balanced control; bottom-right 2 for FAMOS control.

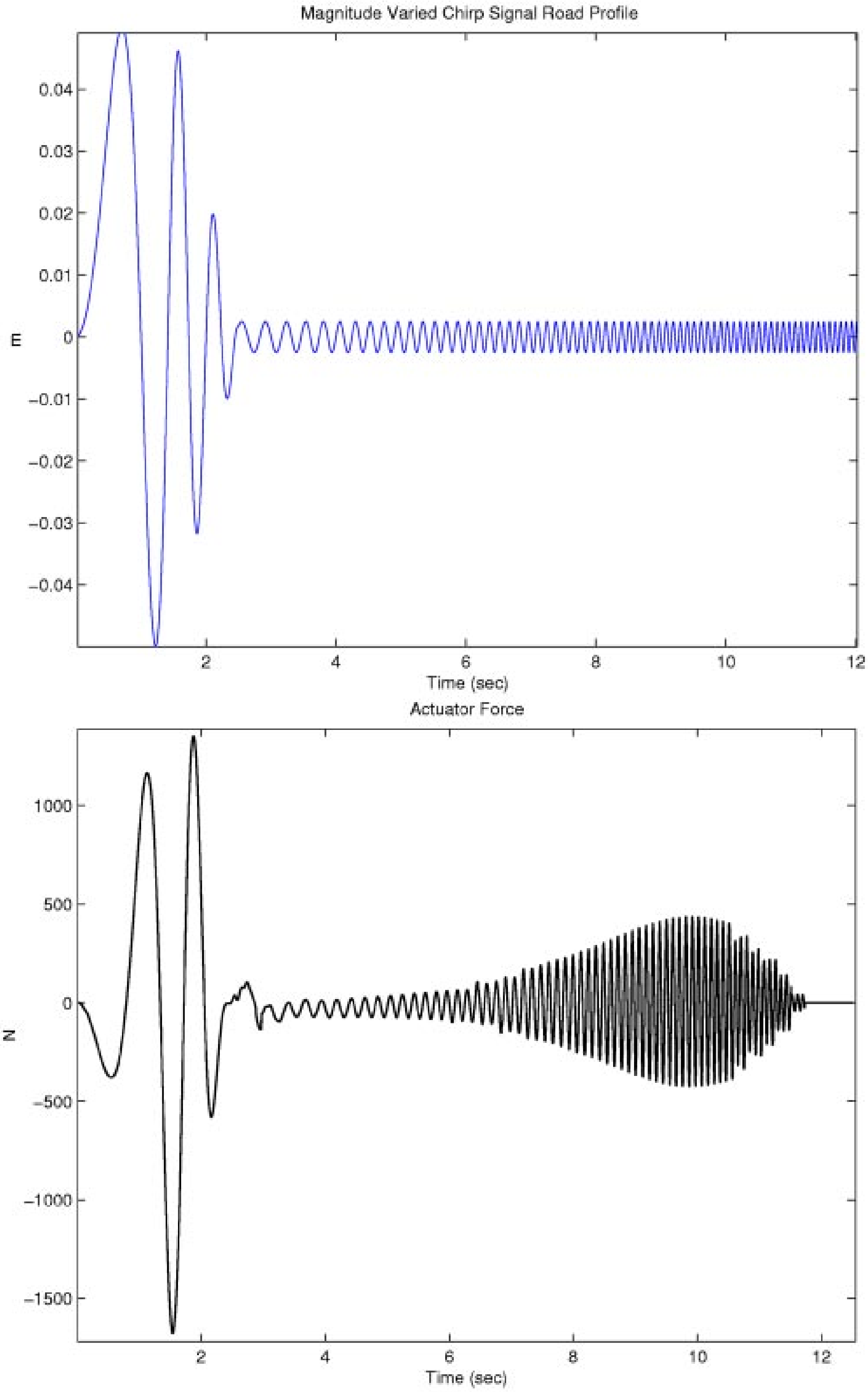

Fig. 5 Top: the magnitude-varied chirp signal with frequency from 0 to 12.5 Hz. Bottom: the corresponding time responses of the suspension control force using FAMOS.

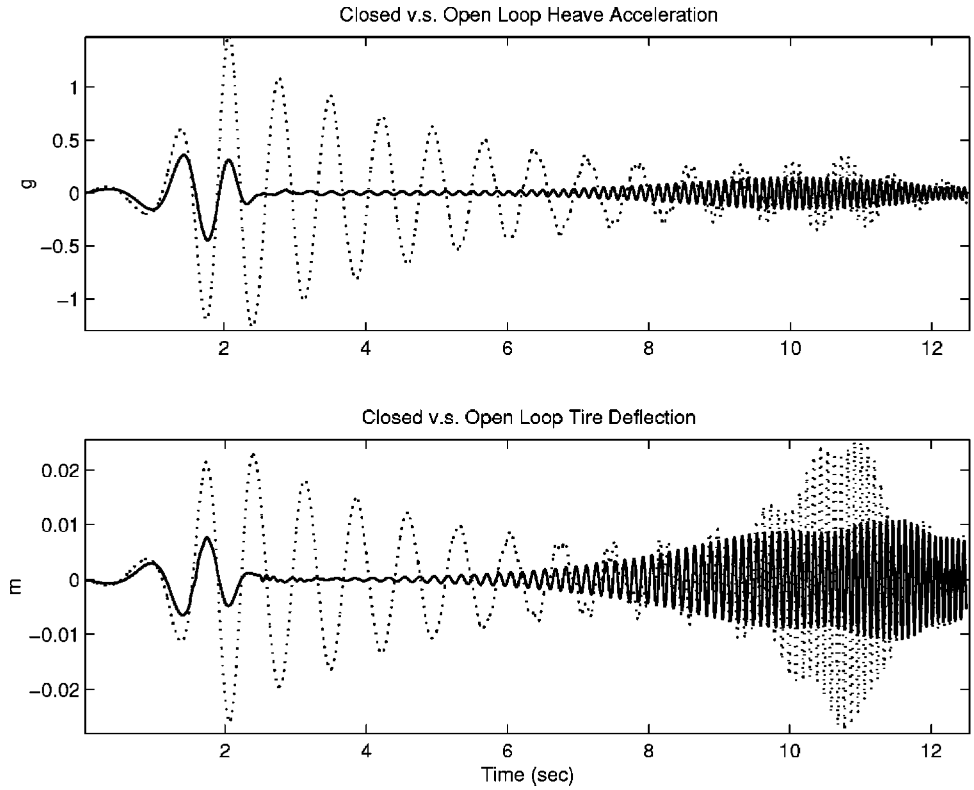

Fig. 6 Time responses with respect to magnitude-varied chirp road profile. Dotted line: passive. Solid line: FAMOS.

$$B_1=\begin{bmatrix}0\\-1\\0\\0\end{bmatrix},\quad B_2=\begin{bmatrix}0\\0\\-\dfrac{1}{m_b}\\-\dfrac{1}{m_w}-\dfrac{1}{m_b}\end{bmatrix}$$

$$C_y=[1\ \ 0\ \ 0\ \ 0],\quad D_{y1}=0,\quad D_{y2}=0$$

$$C_2=\left[-\frac{k_s}{m_b}\ \ 0\ \ 0\ \ -\frac{c_s}{m_b}\right],\quad D_{21}=0,\quad D_{22}=-\frac{1}{m_b}$$

$$C_\infty=[0\ \ 0\ \ 1\ \ -1],\quad D_{\infty 1}=-1,\quad D_{\infty 2}=0.$$

Three controllers are obtained based on three different γs. For large γ but less than 1, the controller tries to reduce the H_2 norm for the body heave acceleration. We should call this the BRP-dominated controller. This controller must guarantee that the body heave acceleration is reduced to below the passive suspension's level within frequency regions Ω_b and Ω_m. Several trials have been conducted through varying γ and finally a BRP-dominated controller is found corresponding to $\gamma=0.67$. We denote this controller as C^b. The top-left two figures in Fig. 4 show the frequency responses of the heave acceleration and the tire deflection with respect to the road profile velocity. Several interested features can be seen from these responses. First, the controller reduces the heave frequency response magnitude to below the passive suspension case (in dotted line) for both Ω_b and Ω_m frequency regions. At part of the wheel frequency region (up to 9.5 Hz), the body performance is worse than the passive suspension case. The controlled suspension continues improving body performance over passive suspension from 9.5 to 11.5 Hz. Then the body performance gets worse again in the harshness frequency region Ω_h. Due to worse BRP in frequency regions Ω_w and Ω_h, the designed controller did not really achieve the desired BRP. However, it is probably close to the best that a single linear controller can achieve due to performance limitation. For the wheel performance, the peak value of the tire deflection frequency response is reduced in wheel frequency region Ω_w. Notice that the peak frequency response is shifted a little bit toward lower frequency due to the active control. The achieved H_∞ norm for the tire deflection is

$$\|T_\infty(P,C^b)\|_\infty=0.5701\|T_\infty(P,0)\|_\infty.$$

For small γ, the corresponding controller emphasizes more on reducing the H_∞ norm for the tire deflection. We should call this WFP-dominated controller. This controller must guarantee that the tire deflection peak frequency response is significantly reduced in wheel frequency region Ω_w. By varying γ, we find one of such controllers corresponding to $\gamma=0.24$, denoted as C^w. The top-right two figures in Fig. 4 show the frequency responses for the body heave acceleration and the tire deflection with respect to the road profile velocity. This controller achieves good WFP in the sense that the tire deflection frequency response is almost flat in wheel frequency region Ω_w with significant peak reduction over passive suspension. However, the body acceleration frequency response gets worse than the passive suspension in the middle frequency region Ω_m, part of the wheel frequency region Ω_w and the whole harshness frequency region Ω_h. It is not hard to conclude that good wheel performance is achieved at the expense of body performance. This result is consistent with the nature of the WFP-dominated controller. The achieved H_∞ norm is

$$\|T_\infty(P,C^w)\|_\infty=0.2155\|T_\infty(P,0)\|_\infty.$$

For a γ in between 0.24 and 0.67, the corresponding suspension controller delivers a closed loop system with frequency response in between those of BRP and WFP. One such controller corresponding to $\gamma=0.38$ is denoted as C^{bw}. The bottom-left two figures of Fig. 4 show the frequency responses using C^{bw}. For this controller BRP is achieved up to 6.5 Hz, and the high frequency magnitudes in body heave acceleration are higher than the BRP-dominated controller but less than the WFP-dominated controller. The wheel performance shows similar trends. The achieved H_∞ norm is

$$\|T_\infty(P,C^{\mathrm{bw}})\|_\infty = 0.3278\|T_\infty(P,0)\|_\infty .$$

A FAMOS strategy switches the final controller among C^b, C^w and C^{bw}, which is implemented in Simulink. Since the controller C^b achieves BRP in Ω_b and Ω_m, FAMOS chooses the BRP-dominated controller up to 8 Hz (see the bottom-right two figures of Fig. 4). After 8 Hz, this controller is gradually faded out to zero. If in this case there is no other controller switched in, the frequency response will approach the passive suspension case. However, for achieving WFP, the WFP-dominated control is gradually scheduled in after 8 Hz. That is, in Ω_w, the tire deflection frequency response peak is reduced. After the frequency identifier shows road frequency contents above 12 Hz, the suspension controller is gradually switched to zero. The time responses of the body heave acceleration and tire deflection with respect to a magnitude-varied chirp signal road profile with frequency up to around 12.5 Hz (see the top figure of Fig. 4) is shown in Fig. 6. The actuator force is shown in the bottom figure of Fig. 5. The body heave acceleration response in all the frequency regions is reduced and the peak value of the tire deflection in wheel frequency region is also reduced significantly. This FAMOS strategy achieves the desired control performance.

6 Conclusion

This paper investigates a marriage between a frequency-adaptive approach and a multi-objective control approach. One of the important enablers for such an approach is that the road profile (or road disturbance) can be identified in real time from the sensor signals. This FAMOS strategy first finds several multi-objective controllers based on different performance preferences, and then switches among those multi-objective controllers based on the identified frequency contents of the detected road profile velocity. The simulation results show improved suspension performances.

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