Proceedings of the 1997 ASCE Structures Congress,
Portland, Oregon, April 13-16, 1997.

Covariance Control Using Closed Loop Modeling for Structures[1]

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Abstract

This paper presents a low order controller design method, using closed loop modeling plus covariance control, with application to the benchmark problem in structural control for the active mass drive system at the University of Notre Dame [1]. This method finds a satisfactory controller by iterating between closed loop modeling and covariance control. The closed loop modeling implies that the model used for model-based control design is extracted from the feedback system of the last iteration.

Introduction

It is well-known that the modeling and control are not independent problems [2], especially when the performance is stringent and the system is complex. Open loop modeling (identification, model reduction, etc.) may or may not provide a good model for control design. Civil structures are typically large and dynamically rich (complex systems with many vibrational modes). Wise use of the relatively large control energy and the limited control complexity demands a theory for synthesizing a simple controller to achieve relatively stringent performance. In this case, both structure-control interaction and model-control interaction are not negligible. In order to achieve stringent performances, we have to take those interactions into account. Combining closed loop modeling with covariance control [3,4] presents a way to incorporate those interactions. This combination iterates between the control design and plant model extraction from the previous closed loop system model. Due to the closed loop feature, this combination indirectly handles those difficulties raised in structural control: spillover, limited control authority and modeling error.

The benchmark problem in structural control [1] requires designing a compensator of limited complexity, based on a high-fidelity structure model, to achieve as stringent performance as possible. Hence the model-control interaction in the benchmark problem could be very strong, which implies that the combination of the closed loop modeling and control is demanded.

Control Design Problem

The active mass drive system at the University of Notre Dame described in [1] can be depicted by the block diagram shown in (a) of Fig. 1. Where $\ddot{x}_g$ is the ground acceleration (modeling earthquake excitation), w represents the sensor noise. v_a and v_s are signals used for modeling purpose. The system measurements are $y = [y_1^T \ \ \ddot{x}_g]^T$ with $y_1 = [x_m \ \ \ddot{x}_{a_1} \ \ \ddot{x}_{a_2} \ \ \ddot{x}_{a_3} \ \ \ddot{x}_{a_m}]^T$, $x_m, \ddot{x}_{a_1}, \ddot{x}_{a_2}, \ddot{x}_{a_3}$ and $\ddot{x}_{a_m}$ have the same definitions as in [1]. P is a high fidelity linear approximation of the actual plant, which includes

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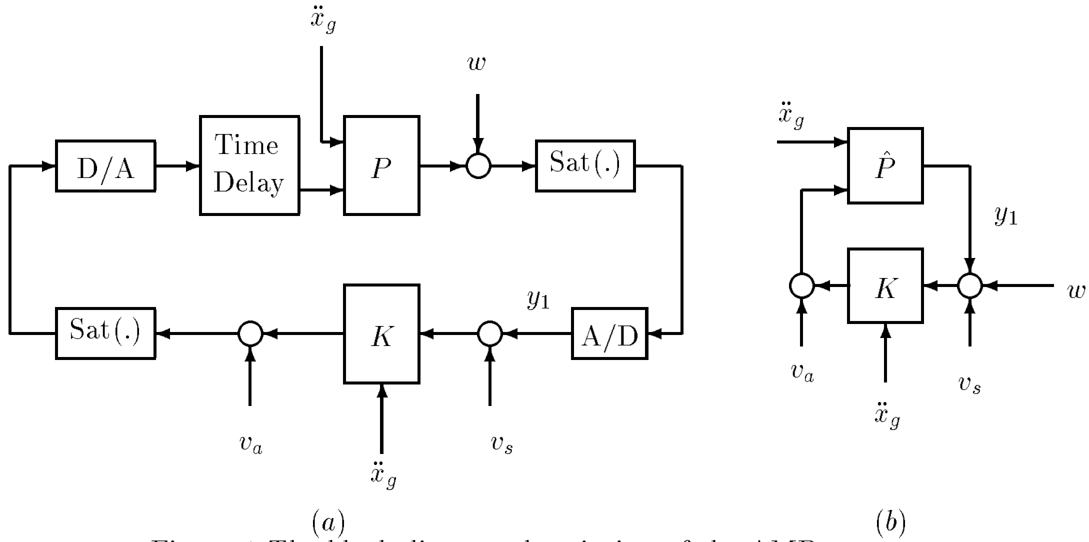

Figure 1 The block diagram description of the AMD system.

possible sensor and actuator dynamics. The state space description of P is

$$\begin{aligned} \dot{x} &= Ax + Bu + E\ddot{x}_g \\ y_1 &= C_y x + D_y u + F_y \ddot{x}_g + w \\ z &= C_z x + D_z u + F_z \ddot{x}_g \end{aligned}$$

K represents the control computer. Considering that the sensor noise w is very small, the diagram in (a) of Fig. 1 can be simplified as the one in (b) of Fig. 1, where $\hat{P}$ is an augmented system including saturation nonlinearities, time delay, A/D and D/A effects. The control problem is to find a discrete time controller of dimension less than 12, with sampling time $T_s = 0.001$ second, to make those performance indices $J_1 \sim J_{10}$ (defined in [1]) as small as possible while maintain some hard limits and robustness requirements. In this paper, the controller for which we are searching is a linear compensator with the following form

$$x_{ck+1} = A_c x_{ck} + B_c(y_k - D_y u_k), \; u_k = (I + D_c D_y)^{-1}(C_c x_{ck} + D_c y_k) \tag{1}$$

where A_c, B_c, C_c, D_c are controller parameters to be determined, denote them as K. With abuse use of notation, we use K to denote the control computer and these control parameters.

Closed Loop Modeling

For a given controller K, a linear model of the augmented plant $\hat{P}$ can be extracted from the identified closed loop system.

Let $\ddot{x}_g$, w, v_a and v_s be white noise sequences with specified covariances. The corresponding output sequence is y_1. From this I/O data pair, a linear approximation of the closed loop transfer matrix $T(s)$ from $[\ddot{x}_g \quad w + v_s \quad v_a]$ to $y_1 + v_s + w$ can be obtained by using the Q-Markov Cover algorithm [3,4]

$$y_1 + v_s + w = T(s) \begin{bmatrix} \ddot{x}_g \\ v_a \\ v_s \end{bmatrix} = \begin{bmatrix} T_g(s) & T_a(s) & T_s(s) \end{bmatrix} \begin{bmatrix} \ddot{x}_g \\ v_a \\ v_s \end{bmatrix}$$

Let $\stackrel{\mathrm{SSR}}{=}$ be short for *state space realization* and the state space realization of the closed loop system T be

$$T \stackrel{\mathrm{SSR}}{=} \begin{bmatrix} \mathsf{D} & \mathsf{C} \\ \mathsf{B} & \mathsf{A} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \mathsf{D}_g & \mathsf{D}_a & \mathsf{D}_s \\ \mathsf{B}_g & \mathsf{B}_a & \mathsf{B}_s \end{bmatrix} & \begin{matrix} \mathsf{C} \\ \mathsf{A} \end{matrix} \end{bmatrix}$$

i.e., the states of T satisfy $\dot{x} = \mathsf{A}x + \mathsf{B}u$, $y = \mathsf{C}x + \mathsf{D}u$. Then the state space realization for $\hat{P}$ is

$$\hat{P} \overset{\mathrm{SSR}}{=} \begin{bmatrix} \mathsf{D}_s & 0 \\ \mathsf{B}_s & I \end{bmatrix}^{-1} \begin{bmatrix} \begin{bmatrix} \mathsf{D}_g & \mathsf{D}_a \\ \mathsf{B}_g & \mathsf{B}_a \end{bmatrix} & \begin{matrix} \mathsf{C} \\ \mathsf{A} \end{matrix} \end{bmatrix} \tag{2}$$

Compensator Design

Consider the model $\hat{P}$ obtained from the closed loop modeling in the last section

$$\begin{aligned} x_{k+1} &= \hat{A}x_k + \hat{B}u_k + \hat{E}\ddot{x}_{gk} \\ z_k &= \hat{C}_z x_k + \hat{D}_z u_k + \hat{F}_z \ddot{x}_{gk} \\ y_{1k} &= \hat{C}_y x_k + \hat{D}_y u_k + \hat{F}_y \ddot{x}_{gk} + w_k \end{aligned} \tag{3}$$

where x represents the system state, z denotes the performance variable, and u is the control variable. The measurement y_1 and w.

Denote $\mathcal{K}$ as the set of all controllers which (i) stabilizes the actual AMD plants; (iii) satisfies the loop gain constraint; (iii) makes the closed loop variables meet the following hard constraints for the ground excitation with Kanai-Tajimi spectrum

$$\mathbf{E}_\infty\, u^2 \leq 1 \text{ volts},\ \mathbf{E}_\infty\, \ddot{x}_{a_m}^2 \leq 2 \text{ g},\ \ \mathbf{E}_\infty\, x_m \leq 3 \text{ cm} \tag{4}$$

and for the 1940 El Centro and 1986 Hachinohe historical earthquake records

$$|u_k| \leq 3 \text{ volts}, \qquad |\ddot{x}_{a_m}| \leq 6 \text{ g}, \qquad |x_{mk}| \leq 9 \text{ cm}. \tag{5}$$

The benchmark problem is actually a multiobjective control problem which can be expressed as

$$\min_{K \in \mathcal{K}} J_i, \quad i = 1, 2, \cdots, 10. \tag{6}$$

Due to the performance criteria J_i's involve specific disturbance sources (historical earthquake records and disturbances with Kanai-Tajimi spectrums) and the hard constraints, there are no systematic methods to exactly solve the above problem. Instead, we model the earth quake disturbances as white noises and solve the following problem

$$\min_{K \in \mathcal{K}} \mathbf{E}_\infty\, z_j^2 \tag{7}$$

where z_j's reflect the variables involved in computing J_i's. In this paper, due to the closed loop modeling feature, we only take those measured variables, i.e., we have $z = [x_m \ \ \ddot{x}_{a_1} \ \ \ddot{x}_{a_2} \ \ \ddot{x}_{a_3} \ \ \ddot{x}_{a_m} \ \ u]^T$. A solvable control problem which reflects indirectly the objectives in (6) or (7) can be further cast into the following constrained optimization

$$\min_K \{\mathbf{E}_\infty\, z_0^T R z_0 \ : \ \mathbf{E}_\infty\, z_1^2 \leq \bar{Z}_1,\ \mathbf{E}_\infty\, z_2^2 \leq \bar{Z}_2,\ \cdots, \mathbf{E}_\infty\, z_{n_z}^2 \leq \bar{Z}_{n_z}\} \tag{8}$$

where n_z is the dimension of z. z_0 is a vector performance variable of the following form $z_{0k} = \hat{C}_0 x_k + \hat{D}_0 u_k + \hat{F}_0 \ddot{x}_g$. this is similar to the so called output variance control (OVC) problem [6]. Notice that a deterministic interpretation of the variance constraint is the peak value constraint.

The above consideration leads to our approach for the benchmark problem, which can be summarizes as the follows: solving the optimization problem (6) indirectly (i) by tuning $\bar{Z}_1, \bar{Z}_2, \cdots, \bar{Z}_{n_z}$ and solving the optimization problem in (8) which takes care of the stabilization and hard constraints; (ii) by incorporating closed loop modeling with (8) which takes care of the control order limitation; (iii) by simulation through the high fidelity evaluation model which finally validates the controller.

Let the covariance of $\ddot{x}_g^T$ and w be W_g and W. For the given variance bounds $\bar{Z}_1, \bar{Z}_2, \cdots, \bar{Z}_{n_z}$, the following generalized output variance control (GOVC) algorithm finds a controller (1) solving (8). The reason that we call the constrained optimization problem (8) generalized output variance control problem is due to: (i) GOVC generalizes the so called OVC problem [6,7], where $z_0 = u$ and z is a linear combination of the plant states and do not include the control variable u; (ii) GOVC deals with generalized plant description of the form (3) and the controller found in the GOVC algorithm is not limited to be strictly proper.

Generalized Output Variance Control Algorithm:

Step 1 Solve for X from the following Riccati equation

$$X = \hat{A}X\hat{A}^T + \hat{E}W_g\hat{E}^T - (\hat{C}_y^T + \hat{E}W_g\hat{F}_y^T)\Psi^{-1}(AX\hat{C}_y^T + \hat{E}W_g\hat{F}_y)^T$$

where $\Psi = \hat{C}_yX\hat{C}_y^T + \hat{F}_yW_g\hat{F}_y^T$. Compute the control parameter

$$B_c = (\hat{A}X\hat{C}_y^T + \hat{E}W_g\hat{F}_y)(\hat{C}_yX\hat{C}_y^T + \hat{F}_yW_g\hat{F}_y^T)^{-1}.$$

Step 2. Choose an initial $Q_0 = \text{diag}(q_{01}, q_{02}, \cdots, q_{0n_z}) > 0$ and compute Y from the following Riccati equation

$$\begin{aligned} Y &= \hat{A}^TY\hat{A} + \hat{C}_0^TR\hat{C}_0 + \hat{C}_z^TQ_0\hat{C}_z - \\ &\quad (\hat{A}^TY\hat{B} + \hat{C}_0^T\hat{D}_0 + \hat{C}_z^TQ_0\hat{D}_z)\Phi^{-1}(\hat{A}^TY\hat{B} + \hat{C}_0^TR\hat{D}_0 + \hat{C}_z^TQ_0\hat{D}_z)^T \end{aligned}$$

where $\Phi = \hat{B}^TY\hat{B} + \hat{D}_0^TR\hat{D}_0 + \hat{D}_z^TQ_0\hat{D}_z$. Compute the control parameters

$$\begin{aligned} &D_c = -\hat{B}^TY\hat{B}\Phi^{-1}, \ \ C_c = -\Phi^{-1}(\hat{A}^TY\hat{B} + \hat{C}_0^TR\hat{D}_0 + \hat{C}_z^TQ_0\hat{D}_z)^T \\ &A_c = \hat{A} + \hat{B}C_c - B_c\hat{C}_y - \hat{B}D_c\hat{C}_y \end{aligned}$$

Step 3. Compute X_c by solving the following Lyapunov equation

$$X_c = (\hat{A}+\hat{B}C_c)X_c(\hat{A}+\hat{B}C_c)^T + (\hat{B}D_c+B_c)(\hat{C}_yX\hat{C}_y^T + \hat{F}_yW_g\hat{F}_y^T + W)(\hat{B}D_c+B_c)^T$$

Step 4. Compute the output covariance of z

$$\begin{aligned} Z &= (\hat{C}_z + \hat{D}_zD_c\hat{C}_y)X(\hat{C}_z + \hat{D}_zD_c\hat{C}_y)^T + (\hat{C}_z + \hat{D}_zC_c)X_c(\hat{C}_z + \hat{D}_zC_c)^T \\ &\quad (\hat{F}_z + \hat{D}_zD_c\hat{F}_y)W_g(\hat{F}_z + \hat{D}_zD_c\hat{F}_y)^T \end{aligned}$$

Let Z_{ii} be the i-th diagonal element of Z. For a given integer β (which affects the convergence rate of the algorithm), compute $q_i = (Z_{ii}/\bar{Z}_i)^\beta q_{i0}$, if $\sum_{i=1}^{n_z} |q_i - q_{0i}| \leq \epsilon$, stop. Otherwise, set $q_i \rightarrow q_{i0}$ and go to step 2, where ϵ is the error tolerance.

Integration of Closed Loop Modeling and Control

The following is the procedure we used to find a satisfactory controller.

Step 1 Let $\bar{Z}_i$ for $i = 1, 2, \cdots, n_z$ be the output variance bounds. Choose integer q (number of Markov/covariance parameters to be matched) and integer n_d (length of the experimental data). Set $i = 0$ and $\hat{P}_0$ as the evaluation model.

Step 2 *GOVC Controller Design*: Do model reduction for $\hat{P}_i$ to obtain a lower order model $\hat{P}_{ir}$. Choose variance bound κ_j, $j = 1, 2, \cdots, n_z$ for the design model $\hat{P}_{ir}$ and design a controller K_i by using the GOVC algorithm. Store the weight Q.

Step 3 *Performance Study*: Evaluate the controller K_i with the evaluation model by white noise excitation and compute the output variances. If the closed loop system is unstable, the design specification κ_i's in step 2 are too tight and must be relaxed. Check whether $\mathbf{E}_\infty z_i^2 \leq \bar{Z}_i$ for all $i = 1, 2, \cdots, n_z$. If this is true, then go to step 5; Otherwise go to step 4 to update the design model.

Step 4 *Closed-loop Modeling*: The state space description for the closed loop system $T_i = [T_g \;\; T_a \;\; T_s]$ can be obtained by using the algorithm presented in [4], which uses the weight Q obtained in step 2. The plant model $\hat{P}_{i+1}$ can be computed from (3). Set $i = i + 1$ and go to step 2.

Step 5 Get the controller formula from previous iteration. **Stop**.

Control Design for Benchmark Problem

By using the procedure to the AMD system, a 10th order controller with measurements $y = [x_m \;\; \ddot{x}_{a_1} \;\; \ddot{x}_{a_2} \;\; \ddot{x}_{a_3} \;\; \ddot{x}_{a_m} \;\; \ddot{x}_g]^T$ is obtained. For the first five criteria, the RMS values of the constraint variables are $\mathbf{E}_\infty\, x_m^2 = 0.4931$ cm, $\mathbf{E}\, \ddot{x}_m^2 = 0.9317$ g, $\mathbf{E}_\infty\, u^2 = 0.1209$ volts. Those satisfy the hard constraints in (4). For evaluation criteria six through ten, the peak values of the constraint variables are $x_m = 2.1157$ cm, $\ddot{x}_m = 5.7748$ g, $u = 0.5933$ volts, which satisfy the hard constraints in (5). The controller achieves the performances summarized in the following table and the loop gain transfer function is shown in Fig. 2.

J_1	J_2	J_3	J_4	J_5
0.2085	0.3162	0.3764	0.3804	0.5205
J_6	J_7	J_8	J_9	J_{10}
0.4417	0.6582	0.7225	0.8912	1.2766

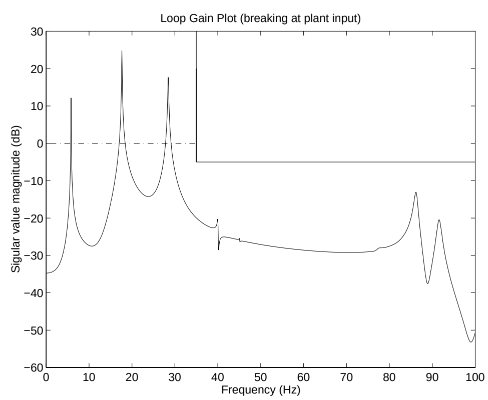

Figure 2: Loop gain transfer function.

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