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Economic Design Problem: Integrating Instrumentation and Control

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Abstract

The so-called signal-to-noise ratio is used to characterize instrumentation, and a system integration is achieved by jointly optimize the feedback control law and the instrument signal-to-noise ratios in order to meet system performance. The procedure identifies the performance limiting components of a control system, identifies where to spend money on a system, and generates component design requirements from closed loop performance. An example is included to illustrate how to use the theory developed here.

1 Introduction

Control system integration should begin with a selection of the components (sensors, actuators, etc.), but theory is not available to decompose the specified closed loop performance into performance requirements of the components. Today most control system designs begins with a selection of component. However, the choice of feedback control law, the selection of variables to be measured and controlled, and the choice of specifications for the given hardware are not independent problems. In order to improve system level performance, there is a need to jointly design the feedback control law and the instrumentation configuration. Instruments may be characterized by their linear dynamic ranges, bandwidths, signal-to-noise ratios etc. In this paper we consider the limitation of signal-to-noise ratios. Although the finite signal-to-noise ratio is a well-known phenomenon, control system design for this model is very new, see [1,2,3,4,5]. The problem studied in this paper can be expressed as the following.

Economic Design Problem:*for a given performance requirement, design the feedback control law and distribute signal-to-noise ratios among the instruments (sensors, actuators, A/D, D/A conversion, control processing) such that the instrumentation cost is minimized without compromising the system performance.*

This paper together with [3] are extracted from [1,2]. For page limitation, the proofs of theorems are omitted. Detailed work can be found in [1,2]. This paper is organized as the follows. Section 2 containes the main results. A numerical example is included in section 3. Section 4 concludes the paper.

2 Integrated Instrumentation and Control

A typical control system can be depicted in Figure 1, where $\hat{P}$ is the plant to be controlled, A and S are the sensor and actuator devices, K is the controller. The augmented plant $P = A\hat{P}S$. Signal a_0 and s_0 are the traditional noises for actuator and sensor devices respectively, where a and s are the internal signals characterizing the effect of finite signal-to-noise ratios (depicted by $\times$ boxes).

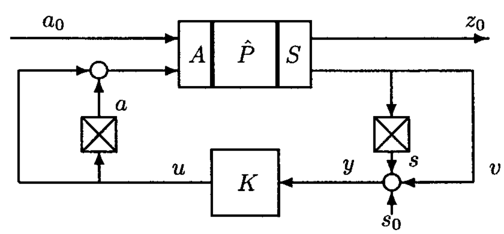

Figure 1: A controlled system with sensor and actuator finite-signal-to-noise model.

For simplicity, let

$$w_0 = \begin{bmatrix} a_0 \\ s_0 \end{bmatrix}, \; w = \begin{bmatrix} a \\ s \end{bmatrix}, \; z = \begin{bmatrix} u \\ v \end{bmatrix}$$
$$\hat{w} = \begin{bmatrix} w_0 \\ w \end{bmatrix}, \; \hat{z} = \begin{bmatrix} z_0 \\ z \end{bmatrix}$$

and partition z and w as

$$w = \begin{bmatrix} w_1^T & w_2^T & \cdots & w_n^T \end{bmatrix}$$
$$z = \begin{bmatrix} z_1^T & z_2^T & \cdots & z_n^T \end{bmatrix}^T.$$

For $i, j = 1, 2, \cdots, n$, denote the closed loop system transfer matrix from w_j to z_i as $T_{z_i w_j}(P, K)$, from $\hat{w}$ to z_i as $T_{z_i \hat{w}}(P, K)$, from $\hat{w}$ to $\hat{z}$ as $T_{\hat{z}\hat{w}}(P, K)$. For notation simplicity, $T_{z_i w_j}(P, K)$, $T_{z_i \hat{w}}(P, K)$ and $T_{\hat{z}\hat{w}}(P, K)$ are short to $T_{z_i w_j}$, $T_{z_i \hat{w}}$ and $T_{\hat{z}\hat{w}}$ respectively.

By generalizing the traditional definition to vector signals, the finite-signal-to-noise effect implies

$$\mathbf{E}_\infty[w_j w_j^T] = R_j\, \mathbf{E}_\infty[\|z_j\|^2], \;\; 0 \le R_j \le \sigma_j I \qquad (1)$$

where $\mathbf{E}_\infty[\cdot]$ denotes the steady state expectation operator; for $j = 1, 2, \cdots, n$ σ_j^{-1} is the signal-to-noise ratio (SNR), while σ_i the noise-to-signal ratio (NSR). The SNR vector and NSR vectors are defined as

$$SNR = \left[\begin{array}{cccc} \sigma_1^{-1} & \sigma_2^{-1} & \cdots & \sigma_n^{-1} \end{array}\right]^T$$
$$NSR = \sigma = \left[\begin{array}{cccc} \sigma_1 & \sigma_2 & \cdots & \sigma_n \end{array}\right]^T$$

We denote all the w whose partitioned components obeying (1) as $\mathcal{W}(\sigma, z)$.

The integrated instrumentation and control problem considered here finds the noise-to-signal ratios associated with the sensors and the actuators and a feedback control law such that the performance level of the controlled variable z_0 can be achieved by the instrumentation configuration with as low cost as possible, which can be summarized as (i) choosing the instrumentation configuration characterized by signal-to-noise ratios; (ii) designing the feedback control logic such that the instrument cost is minimized. The instrument cost is proportional to its signal-to-noise ratio, hence assume that the relative importance of the jth instrument in the total instrument cost is weighted by α_j, then the total instrument cost \$ can be expressed by

$$\$ = \sum_{j=1}^{n} \alpha_j \sigma_j^{-1}.$$

The economic design problem can be further described as solving for the noise-to-signal ratio σ_is and the feedback controller K from the following optimization

$$\min_{\sigma,K}\{\sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \;:\; \max_{w\in\mathcal{W}(\sigma,z)} \mathbf{E}_\infty[\|z_0\|^2] \le \gamma\}. \qquad (2)$$

Using the ϕ measure introduced in [2,4], (2) can be equivalently expressed as

$$\begin{array}{ll} \min_\sigma & \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \\ \text{subject to} & \exists\, K \text{ such that } \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right]) \le 1 \end{array}$$

or further expressed as

$$\begin{array}{ll} \min_\sigma & \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \\ \text{subject to} & \min_K \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right]) \le 1. \end{array} \qquad (3)$$

where

$$\phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right]) = \min_e \max_i \{\frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2 + \sum_{j=1}^{n} \frac{e_j}{e_i}\|T_{z_i w_j}\|_2^2 \sigma_j\}.$$

where

Notice that the minimization of the ϕ-measure in (3) indirectly contributes to reducing the instrumentation cost \$. Assume an optimal controller K, the vector e and the optimal index i for the following optimization

$$\min_K \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right])$$

are given, then

$$\phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right]) = \frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2 + \sum_{j=1}^{n} \frac{e_j}{e_i}\|T_{z_i w_j}\|_2^2 \sigma_j.$$

In this case, (3) can be simplified into the following

$$\begin{array}{ll} \min_\sigma & \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \\ \text{subject to} & \frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2 + \sum_{j=1}^{n} \frac{e_j}{e_i}\|T_{z_i w_j}\|_2^2 \sigma_j \le 1. \end{array} \qquad (4)$$

By using Khun-Tucker multiplier, the optimal solution for (4) occurs at

$$\sigma = \sqrt{\frac{e_i}{\beta}} \left[\begin{array}{ccc} \sqrt{\frac{\alpha_1}{e_1}}\frac{1}{\|T_{z_i w_1}\|_2} & \cdots & \sqrt{\frac{\alpha_n}{e_n}}\frac{1}{\|T_{z_i w_n}\|_2} \end{array}\right]^T.$$

where β is the multiplier and can be computed as

$$\sqrt{\beta} = \frac{\sum_{j=1}^{n} \sqrt{\frac{e_j \alpha_j}{e_i}}\|T_{z_i w_j}\|_2}{1 - \frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2}, \qquad (5)$$

and the optimal instrument cost is

$$\$ = \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} = \frac{\{\sum_{j=1}^{n} \sqrt{\frac{\alpha_j e_j}{e_i}}\|T_{z_i w_j}\|_2\}^2}{1 - \frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2}. \qquad (6)$$

(6) implies that the instrumentation cost is proportional to the gap between the nominal closed loop performance and the performance bound γ. If the gap is large, there are room to choose coarse instruments and if the gap is small more precise instruments must be chosen. For the above constrained optimization, the first desire is to make the constraint feasible, i.e.

$$\min_K \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}\right]) \le 1.$$

The following provides a method to adjust both the noise-to-signal ratio σ and the controller K such that this constraint is kept feasible.

Theorem 2.1: *Consider the following iteration for ϕ_k and σ^k with a given γ and initial noise-to-signal ratio σ^0*

$$\sigma^k = \sigma^{k-1}/\phi_{k-1}, \;\; \phi_{k+1} = \min_K \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma^k \end{smallmatrix}\right]) \qquad (7)$$

$$\phi_0 = \min_K \phi(T_{\hat{z}\hat{w}}, \left[\begin{smallmatrix} \gamma^{-1} \\ \sigma^0 \end{smallmatrix}\right]).$$

If $\phi_0 > 1$, then this iteration converges. Furthermore there exists $\sigma^* \in \mathbb{R}_+^n$ such that

$$\lim_{k\to\infty} \sigma^k = \sigma^*, \quad \lim_{k\to\infty} \phi_k = 1.$$

Proof: See [1,2].

The problem (3) can not be exactly solved by existing algorithms (can not be exactly solved by computationally tractable algorithms). In the following, we consider an iterative algorithm to find a locally optimal solution for (3). This algorithm iterates between solving the following control design problem for an optimal noise-to-signal ratio σ

$$K = \arg\min_K \phi(T_{\hat{z}\hat{w}}, [\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}]), \tag{8}$$

and solving K, σ to enforce the constraint

$$\min_K \phi(T_{\hat{z}\hat{w}}, [\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}]) = 1.$$

The latter can be achieved by using theorem 2.1. Due to the feature of enforcing the ϕ measure and adjusting the noise-to-signal ratio σ, we should call this algorithm the $\sigma - \phi$ iteration. (8) can be solved by the $e - K$ iteration proposed in [2,4]. The following provides a summary of the discrete time version of $e-K$ itteration.

e-K Iteration:

(i) *Set $k = 0$ and choose an initial $e^k \in \mathbb{R}_+^{n+1}$.*

(ii) *Set $e = e^k$ and for this e, solve the following multiobjective H_2 control problem*

$$K_k = \arg\min_K \max_{1\le i\le n, e_i\neq 0} \frac{\|T_{z_i w} W^{\frac{1}{2}}(e)\|_2^2}{e_i}$$

where

$$W(e) \triangleq \text{block diag}(e_1\sigma_1 I, e_2\sigma_2 I, \cdots, e_n\sigma_n I).$$

(iii) *Set $K = K_k$ and for this K, solve the following eigenvalue problem*

$$\phi_k = \max_{1\le i\le n, e_i\neq 0} \frac{\|T_{z_i w} W^{\frac{1}{2}}(e)\|_2^2}{e_i}$$

with e^{k+1} satisfying

$$\sum_{i=1}^n e_i^{k+1} = 1.$$

(iv) *If the relative error at two sequential iterations satisfies*

$$\frac{\|e^{k+1} - e^k\|}{\|e^k\|} + \frac{|\phi_{k+1} - \phi_k|}{\phi_k} < \epsilon$$

then, stop. Otherwise, set $e^k \to e^{k+1}$ and $k = k+1$, go to step (ii). Where ϵ is a given error tolerance.

This $e - K$ iteration converges to a locally optimal solution and the proof can be found in [2,4].

Combining theorem 2.1 and e-K iteration, the $\sigma - \phi$ iteration can be summarized as the following

$\sigma - \phi$ Iteration:

Step 1. *Set $k = 0$. Given a vector σ find a feedback controller K and a vector e to solve*

$$\phi_k = \min_{K, e\in\mathbb{R}^{n+1}} \max_{0\le i\le n, e_i\neq 0} \frac{\|T_{z_i\hat{w}} W_\gamma^{\frac{1}{2}}(e,\sigma)\|_2^2}{e_i}$$

using the $e - K$ iteration. If $\phi_k \not< 1$, conduct iteration in (7) until $\phi_k \le 1 + \epsilon$ and set σ to be this σ^k.

Step 2. *For the given K and e, find the index i such that*

$$\phi_k = \max_i \{\frac{e_0}{e_i\gamma}\|T_{z_i w_0}\|_2^2 + \sum_{j=1}^n \frac{e_j}{e_i}\|T_{z_i w_j}\|_2^2 \sigma_j\}.$$

and compute the noise-to-signal ratio vector from (5), denote it as $\tilde{\sigma}$.

Step 3. *If $\|\sigma - \tilde{\sigma}\| \le \epsilon$, stop; otherwise set $\sigma = \tilde{\sigma}$, $k = k+1$ and go to Step 1.*

3 Example

Let's consider the control system integration for civil structure applications. The civil structure application requires a reliable and cost-effective means of reducing the response of the structure to environmental loads. The key issues in civil structure control include the large power consumption and high instrument cost. The $\sigma - \phi$ algorithm will be used to design a control system with low instrument cost in the lateral vibration control of a 5 story building depicted in Figure 2. Figure 2c represents our simplified model to illustrate the main ideas of this paper. More realistic model will be studied later. This building is subjected to a one-dimensional earthquake excitation $\ddot{x}_g$ at its base and another one dimensional force excitation on the top (to simulate the wind disturbance applied to the building). The natural damping and stiffness might not be enough for the building to adequately suppress the vibrations caused by these disturbances. Hence the active controls are applied. Denote m_i, d_i, k_i as the mass, damping and stiffness coefficients of the i-th floor of the building, then the equation of motion of the building can be expressed as

$$M(\ddot{q} + B_g\ddot{x}_g) + D\dot{q} + Eq = B_2 u + B_t w_t$$

where $q(t) = [q_5(t)\ q_4(t) \cdots q_1(t)]^T$ denotes the floor displacements relative to the ground, and

$$M = \mathrm{diag}(m_5, m_4, \cdots, m_1)$$

$$D = \begin{bmatrix} d_5 & -d_5 & & & \\ -d_5 & d_5+d_4 & -d_4 & & \\ & -d_4 & d_4+d_3 & -d_3 & \\ & & -d_3 & d_3+d_2 & -d_2 \\ & & & -d_2 & d_2+d_1 \end{bmatrix}$$

$$E = \begin{bmatrix} k_5 & -k_5 & & & \\ -k_5 & k_5+k_4 & -k_4 & & \\ & -k_4 & k_4+k_3 & -k_3 & \\ & & -k_3 & k_3+k_2 & -k_2 \\ & & & -k_2 & k_2+k_1 \end{bmatrix},$$

$\ddot{x}_g$ and w_t denote the base acceleration disturbance and the top wind disturbance,

$$B_g = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad B_t = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = I.$$

and we take the spectral densities of $\ddot{x}_q$ and w_t as $S_{\ddot{x}_g} = 1,\ S_{w_t} = 0.05$. In order to control the lateral vibration of the building, we measure the absolute floor accelerations. The sensor output can be expressed as

$$y(t) = \ddot{q} + B_b \ddot{x}_g + v_0 + v.$$

The spectral densities of the external sensor noises are taken as $S_{v_{0_i}} = 0.01,\ i = 1, 2, \cdots, 5$. The control variables are $u_1, u_2, \cdots, u_5$. The normalized parameters for $i = 1, 2, \cdots, 5$ are

$$\begin{aligned} m_i &= 1.1 - 0.1i \\ k_i &= 1.1 - 0.1i \\ d_i &= 0.022 - 0.002i \end{aligned}$$

The instrumentation cost is defined as

$$\text{Instrument cost } \$ = \sum_{i=1}^{10} \alpha_i \sigma_i^{-1}$$

with the weights as $\alpha = \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$, i.e., all the instruments are assumed to have the same price. We consider designing a controller and the signal-to-noise ratios associated with the sensors and actuators to achieve the variance bound for the interstory drift

$$z_{isd}(t) = \begin{bmatrix} 1 & -1 & & & \\ & 1 & -1 & & \\ & & 1 & -1 & \\ & & & 1 & -1 \end{bmatrix} q(t)$$

Let's consider an initial control configuration with a given noise-to-signal ratio vector

$$\sigma = 0.5 \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}. \qquad (9)$$

For this configuration, the instrument cost is

$$\text{Instrument cost } \$ = 20.$$

We want to design a feedback control law together with the instrument noise-to-signal ratios to optimize the instrumentation cost for the performance requirement for z_{isd}.

Let's first design controller K's for the initial instrumentation configuration (9) . The achievable optimal performances for fixed noise-to-signal ratio σ's in (9) will be used as a base performance for integrated design (simultaneously designing K and σ). By conducting $e-K$ iteration and a line search for γ to find a controller and the minimum γ^{base} such that

$$\max_{\text{FSN effect (1)}} \mathbf{E}_\infty[\|z_{isd}\|^2] \le \gamma^{\text{base}}.$$

The $e-K$ iterations for different performance bounds are shown in Figure 3, where each curve terminates at an iteration index which is the total number of iteration needed for the $e-K$ iteration to converge (with an error tolerance $\epsilon = 10^{-3}$). The optimal performance level is the γ value corresponding to $\phi = 1$

$$\gamma^{\text{base}} = 11.5925.$$

Now we are ready to conduct redesigning both the control logic K and the noise-to-signal ratios based on the configuration (9). The goal of the integrated instrumentation and control is to find the noise-to-signal ratios and a control law K such that the worst case output variance of $z_{isd}(t)$ is bounded by γ, i.e.

$$\max_{\text{FSN effect (1)}} \mathbf{E}_\infty[\|z_{isd}\|^2] \le \gamma$$

The performance bounds here are chosen as the following, based on the optimal performances for configuration (9)

$$\gamma = \begin{bmatrix} 0.5 & 1 & 1.5 \end{bmatrix} \gamma^{\text{base}}$$

where $\gamma = 0.5\gamma^{\text{base}}$ corresponds to the most stringent performance requested. With the instrumentation configuration as in (9), there is no control to achieve this performance level. For the stringent performance requirement $\gamma = 0.5\gamma^{\text{base}}$, the $\sigma-\phi$ iteration converges after the 14-th iteration for an error tolerance $\epsilon = 0.005$. The instrument cost corresponding to iteration numbers are shown in Figure 4. Table 1 summarizes the results for the integrated instrumentation and control design. For the stringent performance, we could see from this table that the finest actuator and sensor instrument are placed on the 4th floor. Sensors on the 1st, 2nd and 3rd floors could be very coarse, while the actuator devices on the 2nd and 3rd floors could be very coarse. It is not hard to notice that the control energy is not a monotonic function of the performance.

Table 1: Summary of the integrated design for control objective 1.

	$0.5\gamma_1^{base}$	γ_1^{base}	$1.5\gamma_1^{base}$
Instrumentation cost	53.1388	16.0383	10.0504
Control energy	4.9756	5.4145	3.7248
Sensor SNRs	8.5524 15.9021 4.2376 2.8366 2.3396	1.6500 1.2848 1.0236 2.1506 1.7043	0.8599 0.6796 0.6453 1.3469 1.2605
Actuator SNRs	5.6634 6.5317 1.7072 1.6196 3.7487	1.2732 0.7300 0.9182 1.4845 3.8190	0.7568 0.4317 0.6951 0.8574 2.5217

4 Conclusion

If the financial cost of a component is proportional to its signal-to-noise ratio, then the problem studied here can be interpreted as an "economic" design in the sense that when the optimized precision of a component is small, the component does not need high precision and might be purchased from off-the-shelf hardware, or perhaps the component can be deleted altogether, without a large impact on the system performance. Therefore the solution obtained here identifies where to spend money on a system, beginning from closed loop performance criteria, and generating design requirements for each component of the system. The solution obtained here can also be used to identify the performance limiting components of a closed loop system. This paper provides a first attempt to conduct both theoretic and numerical study for system level integration, which is a small step towards a rather broad integration of control and instrumentation design. Future work will focus on other integrations.

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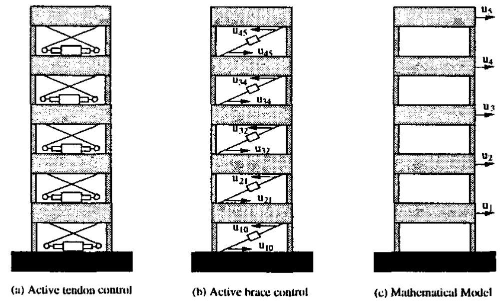

Figure 2: A 5 story building with active control.

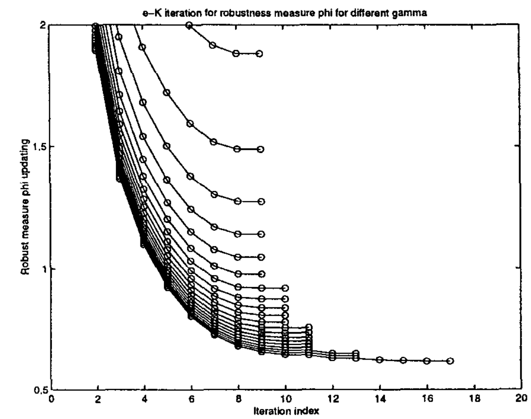

Figure 3: $e - K$ iterations for control objective 1 with instrumentation configuration (9): the eigenvalues converge to ϕ-measures for different performance requirements. Lower ϕ-curves correspond to smaller γ's.

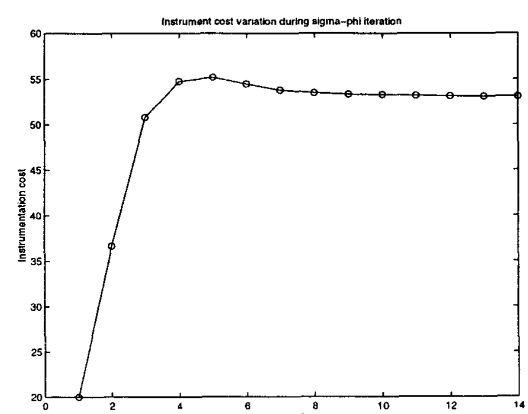

Figure 4: For control objective 1 with stringent performance requirement $\gamma = 0.5\gamma^{base}$, the instrumentation cost during the $\sigma - \phi$ iteration.