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Integrated Instrumentation and Control Design Using Finite Signal-to-noise Models*

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Abstract

In this paper, new signal-to-noise models are used to characterize instrumentation design specifications and an integration of control and instrumentation design is achieved by simultaneously designing the feedback control law and the instrument signal-to-noise ratios to meet performance requirements for the closed loop system. Iterative algorithms are proposed to find locally optimal solutions. Assuming that the signal-to-noise ratio is directly related to the instrumentation cost, this integration provides a systematic procedure to design a low cost control system. More importantly, this procedure identifies the performance limiting component of a closed loop system.

1. Introduction

Control system integration should begin with a selection of the components (sensors, actuators, etc.), but theory is not available to decompose the specified performance of the closed loop system into performance requirements of the components. Many control design methods start after the architecture is specified (sensor/actuator configuration has been set up). However, the choice of feedback control law, the selection of variables to be measured and controlled, and the choice of design specifications for the given hardware are not independent problems. Technology based upon the traditional separation of these problems can not yield performance that is available from joint optimization of control and system parameters. Well-known results show that the system performance might be improved by deleting a noisy actuator, or might be degraded in implementation due to a hardware component with improper design specifications. For example, when performance is not acceptable, it is not clear whether to modify the manufacturing tolerance, the signal processing, the sensor accuracy and the control law. Therefore in order for a designed control system to achieve an optimal performance, a theory is needed to jointly design the feedback control law and the instrumentation configuration.

In this paper instruments are characterized by their signal-to-noise ratios. Although the finite signal-to-noise ratio is a well-known phenomenon in instrumentation, control system design for this model is very new. [4, 1] studied the LQG control problem for a first order systems (the state is a scalar) where the actuator noise variance is linearly related to the actuator signal variance. [5] is the first to introduce the *finite signal-to-noise* model for general linear systems. For fixed signal-to-noise ratios, designing a feedback control algorithm to tolerate the finite-signal-to-noise effect and to achieve suboptimal performance requirements is studied in [3, 6, 7]. [2] proposed a convergent algorithm to synthesize an output feedback controller.

The integrated design of signal-to-noise ratio and feedback control algorithm is studied in this paper. The theoretic issues related to this integration and numerical algorithms will both be addressed here. More specifically, section 2 provides the mathematical modeling for the continuous time finite signal-to-noise model and analysis results. Section 3 contains the main results and algorithms of this paper to conduct the design for both signal-to-noise ratios and the feedback control laws. Section 4 provides an example. Section 5 gives a brief conclusion. All proofs of theorems and the examples are omitted.

The following notations are used. $\mathbb{R}_+$ and $\mathbb{R}$ denote the sets of positive real numbers and real numbers respectively. For matrix operations, $\geq$ means semi-positive definite and $\leq$ means negative-definite. $\|\cdot\|$ is the vector Euclidean norm. $\mathbf{E}[\cdot]$ denotes the usual expectation operator of a stochastic variable, $\mathbf{E}_\infty[\cdot]$ is the steady state value of the expectation of a stochastic process.

2. FSN Mean-square Analysis

Consider the following differential equation

$$dx = Axdt + \sum_{j=0}^{n} B_j dw_j, \quad dz_i = C_i x dt \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state variable. $z_i \in \mathbb{R}^{q_i}$ for $i \in \{0, \cdots, n\}$. $w_j \in \mathbb{R}^{p_j}$, $j \in \{0, \cdots, n\}$ are white noise processes.

Define the following augmented variable

$$\hat{z} = \begin{bmatrix} z_0^T & z^T \end{bmatrix}^T, \quad \hat{w} = \begin{bmatrix} w_0^T & w^T \end{bmatrix}^T,$$

where $z = \begin{bmatrix} z_1^T & z_2^T & \cdots & z_n^T \end{bmatrix}^T$, $w = \begin{bmatrix} w_1^T & w_2^T & \cdots & w_n^T \end{bmatrix}^T$.

Let the $w_j(t)$'s be uncorrelated, white noise processes. Denote $S_{w_j}(j\omega)$ as the spectral density of $w_j(t)$. For the white noise process $w_j(t)$, this spectral density is constant over frequency $S_{w_j}(j\omega) = W_j, \ j \in \{1, \cdots, n\}$. We consider also the case where $w_j(t)$ is white but its spectral density is characterized only by its upper bound $\pi_j I$

$$S_{w_j}(j\omega) \leq \pi_j I, \ \ j \in \{1, \cdots, n\}.$$

In addition if this constant spectral density is proportional to the variance $\mathbf{E}_\infty[\|z\|^2]$ of the signal $z(t)$, then we have the following finite-signal-to-noise model.

Definition 2.1: *Assume w_i in (1) are uncorrelated, white noise processes, we say the system (1) having finite-signal-to-noise model (FSN model) if the spectral density matrix W_i of w_i satisfies*

$$W_j = \mathbf{E}_\infty[\|z_j\|^2]R_j, \ \ 0 \leq R_j \leq \sigma_j I, \ \ j \in \{1, \cdots, n\}.$$

Denote

$$\boldsymbol{R} \triangleq \{\text{block diag}(R_1, \cdots, R_n) \ : \ 0 \leq R_j \leq \sigma_j I, \ j \in \{1, \cdots, n\}\}.$$

and $\mathcal{W}(\boldsymbol{R}, z)$, the set of all FSN processes.

For the above FSN model, the state covariance $X(t)$ satisfies

$$X(t) = e^{At}X(0)e^{A^Tt} + \int_0^t e^{A\tau}[B_0B_0^T + \sum_{j=1}^n \zeta_j B_j R_j B_j^T]e^{A^T\tau}d\tau \quad (2)$$

where ζ_j is the steady state variance of the system signal $z_j(t)$. In the above formula, we assume that ζ_j exists and is finite. It might be possible that ζ_j is not uniquely determined, hence we need to distinguish when ζ_j exists and when it is finite for $j \in \{1, \cdots, n\}$.

Definition 2.2: *The system (1) is said to be FSN stable if for all initial states $x(0)$, the steady state variance $\zeta_i = \mathbf{E}_\infty[\|z_i(t)\|^2]$ exists and is finite for $i \in \{1, \cdots, n\}$.*

For a stochastic system (1), mean-square property is a well-used concept. The following provides a definition for the stability in mean-square sense, which we will call mean square stability (MSS).

Definition 2.3: *The system (1) is mean-square stable (MSS) if for all initial states $x(0)$, the state mean $\mathbf{E}[x(t)]$ converges to zero and the state covariance $X(t) = \mathbf{E}[x(t)x^T(t)]$ converges to a finite value as $t \to \infty$.*

If (1) is mean-square stable, then there exists a $X \geq 0$ and $\zeta_i \in \mathbb{R}_+$ for $i \in \{1, \cdots, n\}$ such that

$$0 = AX + XA^T + B_0B_0^T + \sum_{j=1}^n \zeta_j B_j R_j B_j^T, \ \ \zeta_i = \text{tr}[C_i X C_i^T].$$

If (1) is FSN stable, then there exists $\zeta_i \in \mathbb{R}_+$ for $i \in \{1, \cdots, n\}$ such that

$$\zeta_j = \text{tr}[\int_0^\infty C_i e^{A\tau}\{B_0B_0^T + \sum_{j=1}^n \zeta_j B_j R_j B_j^T\}e^{A^T\tau}C_i^T d\tau] \quad (3)$$

Denote $\zeta \triangleq \begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_n \end{bmatrix}^T$ then from (3) ζ satisfies the following fixed point relationship

$$\zeta = \mathcal{V}(\zeta) \quad (4)$$

for a properly defined function $\mathcal{V}$. The following theorem characterizes the existence conditions for a ζ to satisfy (4) which is further proved to be equivalent to mean-square stability.

Theorem 2.4: *The following statements are equivalent*

(i) *(1) is FSN stable.*

(ii) *A is a stable matrix and $\mathcal{V}(\cdot) \ : \ \mathbb{R}_+^n \to \mathbb{R}_+^n$ is a contraction in $\mathbb{R}_+^n$, i.e., $\forall \ \zeta, \hat{\zeta} \in \mathbb{R}_+^n$,*

$$\|\mathcal{V}(\zeta) - \mathcal{V}(\hat{\zeta})\| < \|\tilde{\zeta}\|$$

for some vector norm $\|\cdot\|$ equipped by $\mathbb{R}^n$.

(iii) *A is a stable matrix . Then*

$$\max_{R \in \boldsymbol{R}} \bar{\lambda}(\mathtt{G}_R) < 1.$$

where the (i,j)-th element of $\mathtt{G}_R$ is $[\mathtt{G}_R]_{ij} = \|T_{z_iw_j}R_j^{\frac{1}{2}}\|_2^2$ and $T_{z_iw_j}$ is the transfer matrix from w_j to z_i for $i, j \in \{1, \cdots, n\}$. Further we have

$$\max_{R \in \boldsymbol{R}} \bar{\lambda}(\mathtt{G}_R) = \bar{\lambda}(\mathtt{G}\,\text{diag}(\sigma)),$$

and we define this quantity as a robustness measure $\phi(T, \sigma) \triangleq \bar{\lambda}(\mathtt{G}\,\text{diag}(\sigma))$, and $\sigma = [sig_1 \ \cdots \ \sigma_n]^T$.

(iv) *(1) is asymptotically stable in mean square sense.*

The above theorem considers the stability problem. Now let's study the performance problem. The performance studied in this paper is the maximum output variance of z_0 over the FSN process $\mathcal{W}(\boldsymbol{R}, z)$. If we denote the performances as J, then

$$J \triangleq \max_{w \in \mathcal{W}(\boldsymbol{R}, z)} \{\mathbf{E}_\infty[\|z_0\|^2] \ : \ (1), \ (4)\}$$

It is not hard to obtain the following relationship by some algebraic manipulations

$$\mathbf{E}_\infty[\|z_0\|^2] = \|T_{z_0w_0}\|_2^2 + \sum_{j=1}^n \|T_{z_0w_j}R_j^{\frac{1}{2}}\|_2^2 \zeta_j \quad (5)$$

if there exists a $\zeta = \begin{bmatrix} \zeta_1 & \zeta_2 & \cdots & \zeta_n \end{bmatrix}^T$ satisfying (4).

The following theorem characterizes the conditions to bound the worst case performance J, which can be expressed by using the ϕ-measure defined in theorem 2.4.

Theorem 2.5: *The worst-case performance with respect to all FSN processes in $\mathcal{W}(\boldsymbol{R}, z)$ satisfies the following bound $J < \gamma$ if and only if*

$$\phi(T_{\hat{z}\hat{w}}, \begin{bmatrix} \gamma^{-1} & \\ & \sigma \end{bmatrix}) < 1.$$

3. Instrumentation and Control

Assume that each instrument variable (for example, actuator and sensor variables) corresponds to an instrument device. The plant G including different I/O drive ports used for instruments can be described by the following state space description

$$\begin{aligned} dx &= Axdt + \sum_{j=0}^{n} B_j dw_j + B_u udt \\ dz_i &= C_i xdt + D_{iu} udt \\ dy &= C_y xdt + \sum_{j=0}^{n} D_{yj} dw_j + D_{yu} udt \end{aligned} \tag{6}$$

Assumption

(A1) (A, B_u) is a stabilizable pair and (A, C_y) is a detectable pair in the usual sense.

The feedback control K sought here is of the following form

$$\dot{x}_c = A_c x + B_c y, \quad u = C_c x \tag{7}$$

Denote the closed loop system from w_j to z_i as $T_{z_i w_j}(P, K)$, and from $\hat{w}$ to z_i as $T_{z_i \hat{w}}(G, K)$.

The control system integration objectives here include (i) choosing the instrumentation configuration; (ii) designing feedback control logic. That is, for a given performance requirement, select the hardware specifications for the sensors and actuators together with a feedback control algorithm. In this paper the control system configuration is determined by the instrument finite-signal-to-noise effects, which are characterized by the FSN models. That is, the (w_i, z_i) pair for $i \in \{1, \cdots, n\}$ satisfies the relationships in definition 2.1. The portion of the system without finite-signal-to-noise effect is called the *nominal system* denoted as G_0, which has the following state space description

$$\begin{aligned} dx &= Axdt + B_0 dw_0 + B_u udt \\ dz_i &= C_i xdt + D_{iu} udt \\ dy &= C_y xdt + D_{y0} dw_0 + D_{yu} udt \end{aligned} \tag{8}$$

Assume that the relative importance of the jth instrument in the cost is weighted by α_j, then the total instrument cost \$ can be expressed by

$$\$ = \sum_{j=1}^{n} \alpha_j \sigma_j^{-1}.$$

The control system integration problem solves for the noise-to-signal ratio σ and the feedback controller K from the following optimization

$$\min_{\sigma, K} \{ \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \ : \ \max_{w \in \mathcal{W}(\boldsymbol{R}, z)} \mathbf{E}_\infty [\|z_0\|^2] \leq \gamma \}. \tag{9}$$

Remark: Notice that the performance level γ must be achievable by the nominal plant G_0, i.e., there exists a controller such that for the nominal plant G_0, we have $\mathbf{E}_\infty [\|z_0\|^2] \leq \gamma$.

3.1. Mean-square Stability Case

Let's first consider the integrated design for mean-square stability, i.e., we want to find the instrumentation configuration with lowest instrument cost and a feedback controller K such that the control system is mean-square stable. Mathematically this can be expressed as

$$\min_{\sigma, K} \{ \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \ : \text{ such that the closed loop system is MSS} \}.$$

Using theorem 2.4, this problem can be expressed as

$$\min_{\sigma, K} \{ \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \ : \ \phi(T_{zw}(G, K), \sigma) < 1 \}. \tag{10}$$

Since if σ, K solve (10), there must exist a $\beta \geq 0$ such that σ, K solve the following unconstrained optimization

$$H = \min_{\sigma, K} \{ \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} + \beta \phi(T_{zw}(G, K), \sigma) \}. \tag{11}$$

Therefore in the following discussion, we study the optimization (11) for a given $\beta > 0$. In this optimization the noise-to-signal ratio σ and the feedback control law K depend on each other. As we know, there are many existing theories to synthesize feedback control laws. Constructing the noise-to-signal ratio σ as a function of the controller K might have certain advantage in the numerical computation. The following discussion is along this line of thinking.

Let $L(K)$ and $N(\sigma)$ be such that

$$H = \min_{K} L(K) = \min_{\sigma} N(\sigma)$$

and it is also not hard to see that $N(\sigma)$ can be computed by the ϕ synthesis proposed in [2]. An algorithm for the ϕ synthesis is the so-called $e - K$ iteration (a convergent algorithm). Now let's consider the computation for $L(K)$.

Theorem 3.1: *For a given controller K, the cost function $L(K)$ can be computed as*

$$L(K) = \min_{e\in\mathbb{R}^n_+} \max_{1\leq i\leq n, e_i\neq 0} 2\sqrt{\beta}\sum_{j=1}^{n}\sqrt{\frac{\alpha_j e_j}{e_i}}\|T_{z_i w_j}(G,K)\|_2.$$

i.e.

$$H = \min_{K,e\in\mathbb{R}^n_+} \max_{1\leq i\leq n, e_i\neq 0} 2\sqrt{\beta}\sum_{j=1}^{n}\sqrt{\frac{\alpha_j e_j}{e_i}}\|T_{z_i w_j}(G,K)\|_2. \quad (12)$$

If K^*, e^*, i^* are the corresponding optimal values for the optimization (12), then the optimal noise-to-signal ratios for the optimization (11) can be computed as

$$\sigma^* = \sqrt{\frac{e^*_{i^*}}{\beta}}\left[\ \sqrt{\frac{\alpha_1}{e^*_1}}\frac{1}{\|T_{z_{i^*} w_1}(G,K^*)\|_2}\ \cdots \right.$$
$$\left.\cdots\ \sqrt{\frac{\alpha_n}{e^*_n}}\frac{1}{\|T_{z_{i^*} w_n}(G,K^*)\|_2}\right]^T. \quad (13)$$

Theorem 3.2: *$L(K)$ can be computed as the following*

$$L(K) = 2\sqrt{\beta}\bar{\lambda}(\mathtt{H}\sqrt{\mathrm{diag}(\alpha)})$$

where the (i,j)-th element of the matrix $\mathtt{H}$ is $[\mathtt{H}]_{ij} = \|T_{z_i w_j}(G,K)\|_2$, and

$$\alpha = \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_n \end{bmatrix}^T.$$

Let f be the eigenvector associated with the eigenvalue $\bar{\lambda}(\mathtt{H}\sqrt{\mathrm{diag}(\alpha)})$ and satisfying $\sum_{j=1}^n f_j = 1$, then the vector e which solves the optimization

$$L(K) = \min_{e\in\mathbb{R}^n_+} \max_{1\leq i\leq n, e_i\neq 0} 2\sqrt{\beta}\sum_{j=1}^{n}\sqrt{\frac{\alpha_j e_j}{e_i}}\|T_{z_i w_j}(G,K)\|_2$$

satisfies $e_j = f_j^2,\ j\in\{1,\cdots,n\}$.

Instead of finding the globally optimal solution for (12), in the following, an iterative algorithm is proposed to find a locally optimal solution. This iteration uses the results presented in theorem 3.2, and is similar to the so-called $e-K$ iteration proposed in [2], except for the adjustment of the optimal noise-to-signal ratios at each iteration. Due to this fact, we should call this algorithm $\sigma - K$ iteration in comparison with $e-K$ iteration.

$\sigma - K$ Iteration:

Step 1. *Set $k=0$. Given the noise-to-signal ratio vector σ find a feedback controller K to solve*

$$\phi_k = \min_{K,e}\max_{1\leq i\leq n, e_i\neq 0}\frac{\|T_{z_i\hat{w}}(G,K)W_\gamma^{\frac{1}{2}}(e,\sigma)\|_2^2}{e_i}.$$

Step 2. *For the controller K obtained in step 1, find the eigenvector f associated with the eigenvalue $\bar{\lambda}(\mathtt{H}\sqrt{\mathrm{diag}(\alpha)})$ such that $\sum_{j=1}^n f_j = 1$ and find the index i such that*

$$\bar{\lambda}(\mathtt{H}\sqrt{\mathrm{diag}(\alpha)}) = \sum_{j=1}^{n}\frac{\sqrt{\alpha_j}f_j}{f_i}\|T_{z_i w_j}(G,K)\|_2.$$

Step 3. *Compute with respect to the vector f and the index i found in step 2*

$$\tilde{\sigma} = \frac{f_i}{\sqrt{\beta}}\left[\frac{\sqrt{\alpha_1}}{f_1}\frac{1}{\|T_{z_i w_1}(G,K)\|_2}\ \cdots\ \frac{\sqrt{\alpha_n}}{f_n}\frac{1}{\|T_{z_i w_n}(G,K)\|_2}\right]^T$$

Step 4. *If $\|\sigma - \tilde{\sigma}\| \leq \epsilon$, stop; otherwise set $\sigma = \tilde{\sigma}$, $k = k+1$ and go to Step 1.*

Theorem 3.3 *The above $\sigma - K$ iteration is a convergent algorithm.*

Designing the control law and the instrumentation configuration to achieve mean-square stability is not of much practical significance due to the fact that a system maintaining mean-square stability could have very bad system performance. However the discussion here shows the hardship of solving the integrated instrumentation and control problem. In the next subsection, we study the integrated design for mean-square performance.

3.2. Robust Performance Case

By using theorem 2.5, the integrated design problem for mean-square performance defined in (9) can be equivalently expressed as

$$\begin{array}{ll}\min_\sigma & \sum_{j=1}^{n}\alpha_j\sigma_j^{-1} \\ \text{subject to} & \exists\ K \text{ such that } \phi(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix}\gamma^{-1} & \\ & \sigma\end{bmatrix}) \leq 1\end{array}$$

or further expressed as

$$\begin{array}{ll}\min_\sigma & \sum_{j=1}^{n}\alpha_j\sigma_j^{-1} \\ \text{subject to} & \min_K \phi(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix}\gamma^{-1} & \\ & \sigma\end{bmatrix}) \leq 1. \quad (14)\end{array}$$

Notice that for fixed γ, $\phi^{-1}(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix}\gamma^{-1} & \\ & \sigma\end{bmatrix})$ is proportional to the size of the set $\boldsymbol{R}$, or the noise-to-signal ratio σ. Hence in (14) the minimization of the ϕ-measure **indirectly** contributes to reducing the instrumentation cost

$$\$ = \sum_{j=1}^{n}\alpha_j\sigma_j^{-1}.$$

Assume an optimal controller K, the vector e and the optimal index i for the following optimization

$$\min_K \phi(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix}\gamma^{-1} & \\ & \sigma\end{bmatrix})$$

are given, then (14) can be simplified into the following optimization

$$\begin{aligned} \min_\sigma \quad & \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} \\ \text{subject to} \quad & \frac{e_0}{e_i \gamma} \|T_{z_i w_0}(G,K)\|_2^2 + \\ & \sum_{j=1}^{n} \frac{e_j}{e_i} \|T_{z_i w_j}(G,K)\|_2^2 \sigma_j \leq 1. \quad (15) \end{aligned}$$

Remark: Notice that a necessary condition for the constraint in (15) to be feasible is that $\|T_{z_0 w_0}(G,K)\|_2^2 \leq \gamma$. Hence in order to conduct the integrated design, all the conditions which guarantee the existence of a controller for the nominal plant G_0 need to be carried out here. This is the reason for assumption (A1).

By using Khun-Tucker multiplier, the optimal solution for (15) occurs at

$$\sigma = \sqrt{\frac{e_i}{\beta}} \begin{bmatrix} \sqrt{\frac{\alpha_1}{e_1}} \frac{1}{\|T_{z_i w_1}(G,K)\|_2} & \cdots & \sqrt{\frac{\alpha_n}{e_n}} \frac{1}{\|T_{z_i w_n}(G,K)\|_2} \end{bmatrix}^T$$

where β is the multiplier and can be computed as

$$\sqrt{\beta} = \frac{\sum_{j=1}^{n} \sqrt{\frac{e_j \alpha_j}{e_i}} \|T_{z_i w_j}(G,K)\|_2}{1 - \frac{e_0}{e_i \gamma} \|T_{z_i w_0}(G,K)\|_2^2}, \quad (16)$$

and the optimal instrument cost is

$$\$ = \sum_{j=1}^{n} \alpha_j \sigma_j^{-1} = \frac{\{\sum_{j=1}^{n} \sqrt{\frac{\alpha_j e_j}{e_i}} \|T_{z_i w_j}(G,K)\|_2\}^2}{1 - \frac{e_0}{e_i \gamma} \|T_{z_i w_0}(G,K)\|_2^2}. \quad (17)$$

(17) implies that the instrumentation cost is also proportional to the gap between the nominal closed loop system performance and the performance bound γ. If the gap is large, there are room to choose coarse instruments and if the gap is small more precise instruments must be chosen for a given performance requirement.

In the design situation, K in (15) must be designed. For the above constrained optimization, the first desire is to make the constraint feasible, i.e.

$$\min_K \phi(T_{\hat{z}\hat{w}}(G,K), [\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}]) \leq 1.$$

The following provides a method to adjust both the noise-to-signal ratio σ and the controller K such that this constraint is kept feasible.

Theorem 3.4: *Consider the following iteration for ϕ_k and σ^k with a given γ and initial noise-to-signal ratio σ^0*

$$\begin{aligned} \sigma^k &= \sigma^{k-1}/\phi_{k-1} \\ \phi_{k+1} &= \min_K \phi(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix} \gamma^{-1} \\ \sigma^k \end{bmatrix}), \quad k \in \{1,2,\cdots,\infty\}. \end{aligned}$$

where

$$\phi_0 = \min_K \phi(T_{\hat{z}\hat{w}}(G,K), \begin{bmatrix} \gamma^{-1} \\ \sigma^0 \end{bmatrix}).$$

If $\phi_0 > 1$, then this iteration converges. Furthermore there exists $\sigma^* \in \mathbb{R}_+^n$ such that

$$\lim_{k\to\infty} \sigma^k = \sigma^*, \quad \lim_{k\to\infty} \phi_k = 1.$$

Similar to the mean-square stability case, the problem (14) can not be exactly solved by existing algorithms (can not be exactly solved by computationally tractable algorithms). An iterative algorithm to find a locally optimal solution for (14) has been proposed. This algorithm iterates between solving problem (15) for an optimal noise-to-signal ratio σ and solving K, σ to enforce the constraint

$$\min_K \phi(T_{\hat{z}\hat{w}}(G,K), [\begin{smallmatrix} \gamma^{-1} \\ \sigma \end{smallmatrix}]) = 1.$$

The latter can be achieved by using theorem 3.4. Due to the feature of enforcing the ϕ measure and adjusting the noise-to-signal ratio σ, we should call this algorithm the $\sigma - \phi$ iteration.

4. Conclusion

The algorithm proposed in this paper can simultaneously design the feedback control algorithm and the instrumentation which is reflected by signal-to-noise ratios to achieve locally optimal instrumentation cost for given performance requirements. The instrumentation configurations are not only affected by control objectives but also the required performance level. This paper provides a first attempt to conduct both theoretic and numerical study for the integration of the instrumentation and control.

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